Lecture 3

Salt Harbor, Prisoners’ Dilemmas & Oil Price Game
• Debrief Salt Harbor

• Overview of Prisoners’ Dilemma Situations
  – The first PD example
  – Analysis of static game

• Iterative Prisoners’ Dilemma
  – Holdup (if time)
  – Oil Price Game
Hard Hat or Soft Hat?

What Should it be?
Which is Better?

- **Black Hat/White Hat**: Few early concessions, followed by increasingly larger concessions
- **White Hat/Black Hat**: Generous early concessions, followed by increasingly smaller concessions
- Hilty and Carnevale found that BH/WH is more effective
Prisoners’ Dilemmas
ADAM SMITH
(The Wealth of the Nations, 1776)

“An individual who intends only his own gain, is, as it were, led by an invisible hand to promote the public interest.”

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Prisoners’ Dilemma

A striking example of how individual rationality and group rationality may diverge
“The Drosophila of the social sciences”
The Prisoner’s Dilemma

An important class of non-strictly competitive situations where the best outcome results when the players refrain from trying to maximize his/her own payoff.
Each player has a dominant strategy and the use of these dominant strategies leads to a “bad” outcome (i.e., Non-Pareto-Optimal)
TWO SUSPECTS ARE TAKEN INTO CUSTODY AND SEPARATED. THE DISTRICT ATTORNEY IS CERTAIN THAT THEY ARE GUILTY OF A SPECIFIC CRIME, BUT HE DOES NOT HAVE ADEQUATE EVIDENCE TO CONVICT THEM AT A TRIAL.

HE POINTS OUT TO EACH PRISONER THAT EACH HAS TWO ALTERNATIVES: TO CONFESS TO THE CRIME THE POLICE ARE SURE THEY HAVE DONE, OR NOT TO CONFESS.

IF THEY BOTH DO NOT CONFESS, THEN THE DISTRICT ATTORNEY STATES HE WILL BOOK THEM ON SOME VERY MINOR PUNISHMENT.

IF THEY BOTH CONFESS THEY WILL BE PROSECUTED, BUT HE WILL RECOMMEND LESS THAN THE MOST SEVERE SENTENCE.

BUT IF ONE CONFESES AND THE OTHER DOES NOT, THEN THE CONFESSOR WILL RECEIVE LENIENT TREATMENT FOR TURNING STATE'S EVIDENCE WHEREAS THE LATTER WILL GET "THE BOOK" SLAPPED AT HIM.
Prisoner’s Dilemma Problem Statement

Two suspects:
  • A
  • B

Two Alternatives:
  • Confess => C
  • Don’t Confess => DC
**Prisoner’s Dilemma Payoff Table**

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<td>C</td>
<td>(8, 8)</td>
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<td>DC</td>
<td>(10, 1/2)</td>
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This is a *non-zero* sum game

No matter what A does, B comes out ahead confessing.
No matter what B does, A comes out ahead confessing.

- For A, strategy C dominates DC strategy
- For B, strategy C dominates DC strategy

If each prisoner chooses his/her dominant strategy, they both lose.

Both players would be better off if neither confess
An outcome that is not dominated by any other outcome is called *Pareto optimal*.

Pareto optimal strategy pairs are (DC,DC), (DC,C) and (C,DC)
What if Prisoners Could Communicate?

Prisoners might choose strategy (DC,DC).

- This, however, is *not an equilibrium pair*, since A and B can each do better by making a unilateral change of choice.

- There is incentive to defect, but if both defect, then we are back where we started from.

- Pre-choice communication cannot help in solving the dilemma unless there is some binding force (legal, moral, etc.) that holds the players to their agreement.
Prisoners and Paradoxes

It is “rational” for each player to confess.

There is no strategy that is best in all circumstances.

Problems such as this confuse our notion(s) of rationality.

— Collective or group rationality vs. individual rationality

Forces us to resort to “Extra-rational” or “Meta-rational” notions (e.g., trust, conscience, etc.)
Applying the Prisoner’s Dilemma

Economists use Prisoner’s Dilemma-type problems in analyzing market structures and competitive strategy.

PD-type problems are common in the real world.

PD creates *price rigidity* in oligopolistic markets.

— Firms may be reluctant to change prices for fear of setting off a price war.

Price leadership as a way around the PD.
Iterative Prisoner’s Dilemma

Removes the static nature of the problem.

Allows players to:

– Develop reputations
– Study competitor’s behavior
When Individuals Meet Often: Axelrod

Strategy: A rule that determines the probability of "cooperate" or "defect" as a function of history of interaction.

WHAT STRATEGIES ARE:

Initially viable?
Robust?
Stable?
One Possible Strategy: “Tit for Tat”

“Tit for Tat”:
First move is to cooperate.
Thereafter, mimic the last move of opponent.

Infinite vs. Finite Trials:
In the infinite case, it always pays to cooperate.
Cooperative behavior is profitable in expected value terms, but depends upon the time horizon in question.
Why Does “Tit for Tat” Work?

Because it is nice (!)
Zero-sum myopia, i.e., score envy
Quick to anger, quick to forgive
Value of provocability
Value of clear and consistent strategies
Axelrod’s Genetic Algorithm (1985)

Simulation of evolution

Computer tournament
  • Round Robin (14 entries)
  • 2nd round (62 entries)

WHAT HAPPENED?

TIT FOR TAT
DOMINATED!

Don’t rock the boat! "C" follows CCC
Be provocable! "D" follows CCD
Forget & forgive! "C" after DCC
Accept a rut! "D" after DDD
“Distinguishing Best and Strategic Practices”
Keith Allred  (2000)

Moves to claim value tend to block moves aimed at creating value

Claiming is competitive and assertive

The downside of pursuing a cooperative strategy aimed at creating value is that it often exposes you to exploitation
Best versus Strategic Practices

BEST PRACTICES:
– Those that work well irrespective of what your negotiating counterparts do

STRATEGIC PRACTICES:
– Practices that work well in particular situations with some counterpart responses and poorly in other situations with other responses
In a formal game-theoretic framework, equilibrium strategies are often, but not always “Best Practice”
– Stud Poker bluffing strategies

Prisoners’ Dilemma games are an example of cases where equilibrium strategy choice may not be a “Best Practice”
– Dominance leads to poor outcomes for both parties
– Holdup!
Holdup

Repeated Prisoner’s Dilemma and Monopoly Power

From “Repeated Interaction”

By Adam Brandenburger

HBS 9-793-116 (1992)
Holdup

• Investor A must first decide whether or not to make an investment of $1
• If made, $3 (gross) of economic value is created.
• B then decides how to divide the $3

  – **Divide** the NET pie of $2 equally: then each gets $1
  – **Grab** the $3 so A is $1 out of pocket
Holdup Tree

- What is likely if the game is played once?
- Ad infinitum?  

Brandenburger op. cit.
Time Value of Money

Suppose that future payoffs are discounted for the time value of money

- $r$ is the discount rate
- $w = 1/(1+r)$ is the discount factor

A constant stream of $1$ per time period then has a present value of $1/(1-w) = 1/r$
• **A’s BEST STRATEGY:**

  Begin by investing every time

  If B ever takes this whole pie, refuse to invest thereafter.
• Split the pie yields

\[ \$1(1 + w + w^2 + \ldots) = \frac{1}{1-w} \]

• Take the whole pie once and get nothing after yields $3.

• A jointly beneficial outcome is sustainable if

\[ \frac{1}{1-w} > 3 \quad \text{or} \quad w > \frac{2}{3} \]
What happens if B splits the pie \( n \) times, then grabs?

- **A** gets
  
  \[
  \$1 \cdot (1 + w + w^2 + \ldots + w^n) - 1 \cdot w^{n+1}
  \]
  
The game stops at \( n + 1 \).

- **B** gets
  
  \[
  \$1 \cdot (1 + w + w^2 + \ldots + w^n) + 3 \cdot w^{n+1}
  \]

- **This strategy pays for B** \( \iff \)
  
  \[
  \frac{1 - w^{n+1}}{1 - w} + 3 \cdot w^{n+1} > \frac{1}{1 - w}
  \]

  or \( \frac{1}{1 - w} < 3 \) or \( w < \frac{2}{3} \)

The same as “Take the Whole Pie!”

Brandenburger op. cit.
Ways Around This Dilemma

Coca Cola facilitated investment in bottling plants by awarding bottlers fixed-price contracts for concentrate.

A computer is designed around a specific microprocessor, adoption of a new chip may be jeopardized by a computer manufacturer’s fears of being held up:

- Intel licensed a new design to a second company (up to intro of 80386 micro-processor)
Holdup: Factors that Facilitate Cooperation

Size of pie

- The greater the value to be divided, the greater the incentive to play “tough” BUT future losses from non-cooperation are also greater!

Discount factor (factor not “rate”):

- A higher discount factor $w=1/(1+r)$ means future losses from non-cooperation loom larger
Frequency of interaction:

– More frequent interaction is equivalent to a larger discount factor

– “Maintenance of an exclusive relationship between buyer and seller can increase interaction frequency”, an argument for dealing only with a few longer term suppliers, for example.

– Break down a large transaction into smaller ones to achieve more frequent interaction
Observation lags:
- If choices are observed with a time lag, future losses from non-cooperation are delayed, making cooperation harder!

Noise:
- Noisy signals make it harder to distinguish tough from cooperative behavior, inhibiting cooperation.
Oil Price Game

Analysis of Payoff Table

& Instructions
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Iterative Dominance

- For Alba $20 preferred to $30
- For Batia $20 preferred to $30

$20 Dominates $30

Price Charged by BATIA

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• Now, for Alba, $10 is preferred to $20
• Now, for Batia $10 is preferred to $20

$10 Dominates $20

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