

Quiz For Lecture # 7

You are consultant for a pension fund. The pension fund is allocating money and considers three institutional investors. The first is a stock investor, the second a long-term government and corporate bond investor, the third is a money market manager.

In your function as the consultant you have done your homework and figured out the following track records:

	expected return	standard deviation
Stock investor	20%	30%
Bond investor	12%	15%
Money market manager	8%	-

The pension fund requires 24% return to meet its asset/liability-objectives. The guidelines do not allow borrowing, however you can sell short. You wish to construct a portfolio of only stocks and bonds with an expected return of 24%

What are the appropriate portfolio proportions and the resulting standard deviations?

What reduction in standard deviation could you attain if you were allowed to borrow at the risk free rate of 8%?

BKM ch. 8, p. 241 # 8

Graph

Point Q is the stock/bond combination with mean = 24%. Let w_s be the proportion of stocks and $1 - w_s$ be the proportion of bonds required to achieve the 24% mean, then:

$$24\% = 20 \times w_s + 12 \times (1 - w_s) = 12 + 8w_s$$

$$w_s = 1.50$$

$$1 - w_s = -0.50$$

Therefore you would have to sell short an amount of bonds equal to 0.50 of your total funds, invest 1.50 times your total funds in stocks. The standard deviation of this portfolio would be:

$$\sigma_Q = [1.50^2 \times 900 + (-0.50)^2 \times 225 + 2 \times (1.50) \times (-0.50) \times 45]^{0.50} = 44.87\%$$

If you were allowed to borrow at the risk free rate of 8%, the way to achieve the target expected return of 24% would be to invest more than 100% of your funds in the optimal risky portfolio, moving out along the CAL, to the right of P, up to R, on the graph.

R is the point on the optimal CAL which has the mean of 24%. Using the formula for the optimal CAL we can find the corresponding standard deviation:

$$E(r_c) = 8 + 0.4601 \sigma_c = 24$$

Setting $E(r_c) = 24$ we get $\sigma_c = 34.78\%$, which is considerably less than 44.78%

Graph

What is the portfolio composition of point R on the optimal CAL? The mean of any portfolio along this CAL is:

Where y is the proportion invested in the optimal risky portfolio P and r_P is the mean of that portfolio, which is 15.61%

$$24 = 8 + y(15.61 - 8)$$

$$y = 2.1025$$

This means that for every \$1 of your own funds invested in portfolio p, you would borrow an additional \$ 1.1025 and invest it also in portfolio P.