LECTURE 17
SYNCHRONIZATION WITHOUT LOCKS

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SEQUENTIAL CONSISTENCY
Initially, \( a = b = 0 \).

**Processor 0**

- `mov 1, a ; Store`
- `mov b, %ebx ; Load`

**Processor 1**

- `mov 1, b ; Store`
- `mov a, %eax ; Load`

Q. Is it possible that Processor 0’s %ebx and Processor 1’s %eax both contain the value 0 after the processors have both executed their code?

A. It depends on the *memory model*: how memory operations behave in the parallel computer system.
Sequential Consistency

“[T]he result of any execution is the same as if the operations of all the processors were executed in some sequential order, and the operations of each individual processor appear in this sequence in the order specified by its program.” — Leslie Lamport [1979]

- The sequence of instructions as defined by a processor’s program are *interleaved* with the corresponding sequences defined by the other processors’ programs to produce a global *linear order* of all instructions.
- A LOAD instruction receives the value stored to that address by the most recent STORE instruction that precedes the LOAD, according to the linear order.
- The hardware can do whatever it wants, but for the execution to be sequentially consistent, it must *appear* as if LOAD’s and STORE’s obey some global linear order.
Initially, \(a = b = 0\).

**Example**

Sequential consistency implies that no execution ends with \(\%eax = \%ebx = 0\).
Reasoning about Sequential Consistency

- An execution induces a “happens before” relation, which we shall denote as $\rightarrow$.
- The $\rightarrow$ relation is **linear**, meaning that for any two distinct instructions $x$ and $y$, either $x \rightarrow y$ or $y \rightarrow x$.
- The $\rightarrow$ relation respects **processor order**, the order of instructions in each processor.
- A **LOAD** from a location in memory reads the value written by the **most recent** **STORE** to that location according to $\rightarrow$.
- For the memory resulting from an execution to be sequentially consistent, there must exist such a linear order $\rightarrow$ that yields that memory state.
Mutual Exclusion without Locks
Recall
A critical section is a piece of code that accesses a shared data structure that must not be accessed by two or more threads at the same time (mutual exclusion).

Most implementations of mutual exclusion employ an atomic read-modify-write instruction or the equivalent, usually to implement a lock:
- e.g., xchg, test-and-set, compare-and-swap, load-linked-store-conditional.
Q. Can mutual exclusion be implemented with LOAD’s and STORE’s as the only memory operations?

A. Yes, Theodorus J. Dekker and Edsger Dijkstra showed that it can, as long as the computer system is sequentially consistent.
Peterson’s Algorithm

widget x; //protected variable
bool A_wants = false;
bool B_wants = false;
enum {A, B} turn;

A_wants = true;
turn = B;
while (B_wants && turn==B);
frob(&x); //critical section
A_wants = false;

B_wants = true;
turn = A;
while (A_wants && turn==A);
borf(&x); //critical section
B_wants = false;
Peterson’s Algorithm

Alice

A_wants = true;
turn = B;
while (B_wants && turn==B);
frob(&x); //critical section
A_wants = false;

Bob

B_wants = true;
turn = A;
while (A_wants && turn==A);
borf(&x); //critical section
B_wants = false;

Intuition

• If Alice and Bob both try to enter the critical section, then whoever writes last to turn spins and the other progresses.
• If only Alice tries to enter the critical section, then she progresses, since B_wants is false.
• If only Bob tries to enter the critical section, then he progresses, since A_wants is false.

But we can do better!
Theorem. Peterson’s algorithm achieves mutual exclusion on the critical section.

Proof.

• Assume for the purpose of contradiction that both Alice and Bob find themselves in the critical section together.

• Consider the most–recent time that each of them executed the code before entering the critical section.

• We shall derive a contradiction.
Proof of Mutual Exclusion

Alice

A_wants = true;
turn = B;
while (B_wants && turn==B);
frob(&x); //critical section
A_wants = false;

Bob

B_wants = true;
turn = A;
while (A_wants && turn==A);
borf(&x); //critical section
B_wants = false;
Proof of Mutual Exclusion

### Alice

```c
A_wants = true;
turn = B;
while (B_wants && turn==B);
frob(&x); //critical section
A_wants = false;
```

### Bob

```c
B_wants = true;
turn = A;
while (A_wants && turn==A);
borf(&x); //critical section
B_wants = false;
```

- WLOG, assume that Bob was the last to write to `turn`:
  
  $$\text{write}_A(\text{turn} = B) \Rightarrow \text{write}_B(\text{turn} = A).$$
Proof of Mutual Exclusion

Alice

\begin{verbatim}
A_wants = true;
turn = B;
while (B_wants && turn==B);
frob(&x); //critical section
A_wants = false;
\end{verbatim}

Bob

\begin{verbatim}
B_wants = true;
turn = A;
while (A_wants && turn==A);
borf(&x); //critical section
B_wants = false;
\end{verbatim}

- WLOG, assume that Bob was the last to write to turn:
  \[ write_A(\text{turn} = B) \Rightarrow write_B(\text{turn} = A). \]
- Alice’s program order:
  \[ write_A(A\_wants = true) \Rightarrow write_A(\text{turn} = B). \]
Proof of Mutual Exclusion

Alice

A_wants = true;
turn = B;
while (B_wants && turn==B);
frob(&x);  //critical section
A_wants = false;

Bob

B_wants = true;
turn = A;
while (A_wants && turn==A);
borf(&x);  //critical section
B_wants = false;

• WLOG, assume that Bob was the last to write to turn:
  \[ \text{write}_A(\text{turn} = B) \rightarrow \text{write}_B(\text{turn} = A) \]

• Alice’s program order:
  \[ \text{write}_A(\text{A_wants} = \text{true}) \rightarrow \text{write}_A(\text{turn} = B) \]

• Bob’s program order:
  \[ \text{write}_B(\text{turn} = A) \rightarrow \text{read}_B(\text{A_wants}) \rightarrow \text{read}_B(\text{turn}) \]

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Proof of Mutual Exclusion

Alice

A_wants = true;
turn = B;
while (B_wants && turn==B);
frob(&x); //critical section
A_wants = false;

Bob

B_wants = true;
turn = A;
while (A_wants && turn==A);
borf(&x); //critical section
B_wants = false;

- WLOG, assume that Bob was the last to write to turn: write_A (turn = B) → write_B (turn = A).

- Alice’s program order:
  write_A (A_wants = true) → write_A (turn = B).

- Bob’s program order:
  write_B (turn = A) → read_B (A_wants) → read_B (turn).

- What did Bob read?
  A_wants: true
  turn: A
  } Bob should spin. Contradiction.

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Theorem: Peterson’s algorithm guarantees starvation freedom: While Alice wants to execute her critical section, Bob cannot execute his critical section twice in a row, and vice versa.

Proof. Exercise. ■
RELAXED MEMORY CONSISTENCY
Memory Models Today

- No modern-day processor implements sequential consistency.
- All implement some form of relaxed consistency.
- Hardware actively reorders instructions.
- Compilers may reorder instructions too.
Q. Why might the hardware or compiler decide to reorder these instructions?

A. To obtain higher performance by covering load latency — *instruction-level parallelism*.
Q. When is it safe for the hardware or compiler to perform this reordering?

A. When $a \neq b$.

A'. And there’s no concurrency.
The processor can issue `STORE’s` faster than the network can handle them ⇒ store buffer.
Since a `LOAD` can stall the processor until it is satisfied, loads take priority, bypassing the store buffer.
If a `LOAD` address matches an address in the store buffer, the store buffer returns the result.
Thus, a `LOAD` can bypass a `STORE` to a different address.
House rules:

1. LOAD’s are *not* reordered with LOAD’s.
2. STORE’s are *not* reordered with STORE’s.
3. STORE’s are *not* reordered with prior LOAD’s.
4. A LOAD may be reordered with a prior STORE to a *different* location but *not* with a prior STORE to the *same* location.
5. LOAD’s and STORE’s are *not* reordered with LOCK instructions.
6. STORE’s to the same location respect a *global total order*.
7. LOCK instructions respect a *global total order*.
8. Memory ordering preserves *transitive visibility* ("causality").
x86–64 Total Store Order

House rules:

1. LOAD’s are not reordered with LOAD’s.
2. STORE’s are not reordered with STORE’s.
3. STORE’s are not reordered with prior LOAD’s.
4. A STORE is not reordered with a prior store.
5. LOAD’s are ordered with global total order.
6. MEMORY’s to the same location respect a global total order.
7. LOCK instructions respect a global total order.
8. Memory ordering preserves transitive visibility (“causality”).

Total Store Ordering (TSO) is weaker than sequential consistency.

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Impact of Reordering

Processor 0

1. mov 1, a ;Store
2. mov b, %ebx ;Load

Processor 1

3. mov 1, b ;Store
4. mov a, %eax ;Load
The ordering \( \langle 2, 4, 1, 3 \rangle \) produces \%eax = \%ebx = 0.

\textit{Instruction reordering violates sequential consistency!}
Peterson’s algorithm revisited

- The LOAD’s of `B_wants` and `A_wants` can be reordered before the STORE’s of `A_wants` and `B_wants`, respectively.
- Both Alice and Bob might enter their critical sections simultaneously!
A memory fence (or memory barrier) is a hardware action that enforces an ordering constraint between the instructions before and after the fence.

A memory fence can be issued explicitly as an instruction (x86: mfence) or be performed implicitly by locking, exchanging, and other synchronizing instructions.

The Tapir/LLVM compiler implements a memory fence via the function atomic_thread_fence() defined in the C header file stdatomic.h.*

The typical cost of a memory fence is comparable to that of an L2-cache access.

Alice

```c
A_wants = true;
turn = B;
while (B_wants && turn==B);
frob(&x); //critical section
A_wants = false;
```

Bob

```c
B_wants = true;
turn = A;
while (A_wants && turn==A);
borf(&x); //critical section
B_wants = false;
```

**Memory fences** can restore sequential consistency.

```c
A_wants = true;
turn = B;
atomic_thread_fence();
while (B_wants && turn==B);
frob(&x); //critical section
A_wants = false;
```

```c
B_wants = true;
turn = A;
atomic_thread_fence();
while (A_wants && turn==A);
borf(&x); //critical section
B_wants = false;
```

Well, sort of. You also need to make sure that the **compiler** doesn’t screw you over.
Back in the day, in addition to the memory fence:

- you must declare variables as `volatile` to prevent the compiler from optimizing away memory references;
- you need `compiler fences` around `frob()` and `borf()` to prevent compiler reordering.
The C11 language standard defines its own weak memory model, in which you can control hardware and compiler reordering of memory operations by:

- Declaring variables as `_Atomic`; and
- Using the functions `atomic_load()`, `atomic_store()`, etc. as needed.

Implementing General Mutexes

**Theorem [Burns–Lynch].** Any $n$–thread deadlock–free mutual–exclusion algorithm using only LOAD and STORE memory operations requires $\Omega(n)$ space.

**Theorem [Attiya et al.]:** Any $n$–thread deadlock–free mutual–exclusion algorithm on a modern machine must use an expensive operation such as a memory fence or an atomic compare–and–swap operation.

Thus, hardware designers are justified when they implement special operations to support atomicity.
COMPARE-AND-_SWAP
Memory operations

- LOAD
- STORE
- CAS (*compare-and-swap*)
The *compare-and-swap* operation is provided by the `cmpxchg` instruction on x86-64. The C header file `stdatomic.h` provides CAS via the built-in function

```c
atomic_compare_exchange_strong()
```

which can operate on various integer types.

### Specification

```c
bool CAS(T *x, T old, T new) {
    if (*x == old) {
        *x = new;
        return true;
    }
    return false;
}
```

- Executes atomically.
- Implicit fence.

**Theorem.** An n–thread deadlock–free mutual–exclusion algorithm using CAS can be implemented using $\Theta(1)$ space.

**Proof.**

```c
void lock(int *lock_var) {
    while (!CAS(*lock_var, false, true));
}
```

```c
void unlock(int *lock_var) {
    *lock_var = false;
}
```

Just the space for the mutex itself. ■
Summing Problem

```c
int compute(const X& v);
int main() {
    const int n = 1000000;
    extern X myArray[n];
    // ...

    int result = 0;
    cilk_for (int i = 0; i < n; ++i) {
        result += compute(myArray[i]);
    }
    printf("The result is: %f\n", result);
    return 0;
}
```

Race!
int compute(const X& v);
int main() {
    const int n = 1000000;
    extern X myArray[n];
    mutex L;
    // ...

    int result = 0;
    cilk_for (int i = 0; i < n; ++i) {
        int temp = compute(myArray[i]);
        L.lock();
        result += temp;
        L.unlock();
    }
    printf( "The result is: %f\n", result );
    return 0;
}
Yet all we want is to atomically execute a **LOAD** of \( x \) followed by a store of \( x \).

```c

Yet all we want is to atomically execute a **LOAD** of \( x \) followed by a store of \( x \).
```
Q. Now what happens if the operating system swaps out a loop iteration?

A. No other loop iteration needs to wait. The algorithm is *nonblocking*. 

```c
int result = 0;
cilk_for (int i = 0; i < n; ++i) {
    int temp = compute(myArray[i]);
    int old, new;
    do {
        old = result;
        new = old + temp;
    } while (!CAS(&result, old, new));
}
```
Lock-Free Algorithms
Lock-Free Stack

```c
struct Node {
    Node* next;
    int data;
};

struct Stack {
    Node* head;
};
```

head: 

![Diagram showing a lock-free stack structure with nodes containing the integers 77 and 75 connected to the head node.](image-url)
void push(Node* node) {
    do {
        node->next = head;
    } while (!CAS(&head, node->next, node));
}
void push(Node* node) {
    do {
        node->next = head;
    } while (!CAS(&head, node->next, node));
}
Node* pop() {
    Node* current = head;
    while (current) {
        if (CAS(&head, current, current->next)) break;
        current = head;
    }
    return current;
}
Compare and compare-and-swap

*Compare-and-swap* acquires a cache line in exclusive mode, invalidating the cache line in other caches.

**Result:** High contention if all processors are doing CAS’s to same cache line.

Better way: First read if value at memory location changed before doing CAS, and only do CAS if value didn’t change.
Lock-Free Push and Pop

```c
void push(Node* node) {
    do {
        node->next = head;
    } while (head != node->next || !CAS(&head, node->next, node));
}

Node* pop() {
    Node* current = head;
    while (current) {
        if (head == current &&
            CAS(&head, current, current->next)) break;
        current = head;
    }
    return current;
}
```
Efficient lock-free algorithms are known for a variety of classical data structures (e.g., linked lists, queues, skip lists, hash tables).

In theory, a thread might starve. Because of contention, its operation might never complete. In practice, starvation rarely happens.

*Transactional memory* is revolutionizing this area.
- Allows executing a block of code atomically.

**Practical Issues**
- Memory management.
- Contention.
- The ABA problem.
The ABA Problem
1. Thread 1 begins to pop the node containing 15, but stalls after reading current->next.
1. Thread 1 begins to pop the node containing 15, but stalls after reading current->next.
2. Thread 2 pops the node containing 15.
1. Thread 1 begins to pop the node containing 15, but stalls after reading `current->next`.
2. Thread 2 pops the node containing 15.
3. Thread 2 pops the node containing 94.
1. Thread 1 begins to pop the node containing 15, but stalls after reading `current->next`.
2. Thread 2 pops the node containing 15.
3. Thread 2 pops the node containing 94.
4. Thread 2 pushes the node 7, reusing the node that contained 15.
1. Thread 1 begins to pop the node containing 15, but stalls after reading `current->next`.
2. Thread 2 pops the node containing 15.
3. Thread 2 pops the node containing 94.
4. Thread 2 pushes the node 7, reusing the node that contained 15.
5. Thread 1 resumes, and its CAS succeeds, removing 7, but putting garbage back on the list.
Solutions to ABA

Versioning

- Pack a *version number* with each pointer in the same atomically updatable word.
- Increment the version number every time the pointer is changed.
- Compare–and–swap both the pointer and the version number as a single atomic operation.

Issue

- Version numbers may need to be very large.

Reclamation

- Prevent node reuse while pending requests exist.
- For example, prevent node 15 from being reused as node 7 while Thread 1 still executing.