Thermal Properties of Foams

- Closed cell foams widely used for thermal insulation
- Only materials with lower conductivity are aerogels (tend to be brittle and weak) and vacuum insulation panels
- Low thermal conductivity of foam arises from:
  - low volume fraction of solid
  - high volume fraction of gas with low $\lambda$
  - small cell size suppresses convection and radiation (through repeated absorption and reflection)
- Applications: buildings, refrigerated vehicles, LNE tankers
- Foams also have good thermal shock resistance since coefficient of thermal expansion of foam equals to that of the solid; plus the modulus is much lower ($\epsilon = \alpha \Delta T \quad \sigma = E \alpha \Delta T = \sigma_f$)
  ⇒ used as heat shields
- Ceramic foams used as firebrick — ceramic has high $T$
  - foam - low $\lambda$ - low heat loss
  - low heat capacity - lowers energy to heat furnace to temperature
  - good thermal shock resistance
Thermal conductivity, $\lambda$

- Steady state conduction ($T$ constant with time)

**Fourier Law:**

$$q = -\lambda \nabla T$$

1D

$$q = -\lambda \frac{dT}{dx}$$

$q$ = hect flux [J/(m²/s)]

$\lambda$ = thermal conductivity [W/mK]

$\nabla T$ = temperature gradient

$$\nabla T = i \frac{\partial T}{\partial x} + j \frac{\partial T}{\partial y} + k \frac{\partial T}{\partial z}$$

- Non-steady heat conduction ($T$ varies with time $t$)

$$\frac{\partial T}{\partial \tau} = a \frac{\partial^2 T}{\partial x^2}$$

$a$ = thermal diffusivity = $\frac{\lambda}{\rho C_p}$

$[m^2/s]$  

- Values for $\lambda$, $a$ Table 7.1
### Table 7.1 Thermal conductivities and diffusivities

<table>
<thead>
<tr>
<th>Material</th>
<th>Thermal conductivity $\lambda$ (W/m K)</th>
<th>Thermal diffusivity $a$ (m$^2$/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper (solid)</td>
<td>384$^a$</td>
<td>$8.8 \times 10^{-5}$ $^a$</td>
</tr>
<tr>
<td>Aluminium (solid)</td>
<td>230$^a$</td>
<td>$8.9 \times 10^{-5}$ $^a$</td>
</tr>
<tr>
<td>Alumina (solid)</td>
<td>25.6$^a$</td>
<td>$8.2 \times 10^{-6}$ $^a$</td>
</tr>
<tr>
<td>Glass (solid)</td>
<td>1.1$^a$</td>
<td>$4.5 \times 10^{-7}$ $^a$</td>
</tr>
<tr>
<td>Polyethylene (solid)</td>
<td>0.35$^a$</td>
<td>$1.7 \times 10^{-7}$ $^a$</td>
</tr>
<tr>
<td>Polyurethane (solid)</td>
<td>0.25$^c$</td>
<td></td>
</tr>
<tr>
<td>Polystyrene (solid)</td>
<td>0.15$^a$</td>
<td>$1.0 \times 10^{-7}$ $^a$</td>
</tr>
<tr>
<td>Air</td>
<td>0.025$^a$</td>
<td></td>
</tr>
<tr>
<td>Carbon dioxide</td>
<td>0.016$^a$</td>
<td></td>
</tr>
<tr>
<td>Trichlorofluoromethane (CCl$_3$F)</td>
<td>0.008$^a$</td>
<td></td>
</tr>
<tr>
<td>Oak ($\rho^\prime/\rho_c = 0.40$)</td>
<td>0.150$^a$</td>
<td></td>
</tr>
<tr>
<td>White pine ($\rho^\prime/\rho_c = 0.34$)</td>
<td>0.112$^a$</td>
<td></td>
</tr>
<tr>
<td>Balsa ($\rho^\prime/\rho_c = 0.09$)</td>
<td>0.055$^a$</td>
<td></td>
</tr>
<tr>
<td>Cork ($\rho^\prime/\rho_c = 0.14$)</td>
<td>0.045$^a$</td>
<td></td>
</tr>
<tr>
<td>Polystyrene foam ($\rho^\prime/\rho_c = 0.025$)</td>
<td>0.040$^b$</td>
<td>$1.1 \times 10^{-6}$ $^b$</td>
</tr>
<tr>
<td>Polyurethane foam ($\rho^\prime/\rho_c = 0.02$)</td>
<td>0.025$^b$</td>
<td>$9.0 \times 10^{-7}$ $^b$</td>
</tr>
<tr>
<td>Polystyrene foam ($\rho^\prime/\rho_c = 0.029–0.057$)</td>
<td>0.029–0.038$^d$</td>
<td></td>
</tr>
<tr>
<td>Polysiocyanurate foam, (CFC-11) ($\rho^\prime = 32$ kg/m$^3$)</td>
<td>0.020$^d$</td>
<td></td>
</tr>
<tr>
<td>Phenolic foam, (CFC-11, CFC-113) ($\rho^\prime = 48$ kg/m$^3$)</td>
<td>0.017$^d$</td>
<td></td>
</tr>
<tr>
<td>Glass foam ($\rho^\prime/\rho_c = 0.05$)</td>
<td>0.050$^d$</td>
<td></td>
</tr>
<tr>
<td>Glass wool ($\rho^\prime/\rho_c = 0.01$)</td>
<td>0.042$^d$</td>
<td></td>
</tr>
<tr>
<td>Mineral fibre ($\rho^\prime/\rho_c = 4.8–32$ kg/m$^3$)</td>
<td>0.046$^d$</td>
<td></td>
</tr>
</tbody>
</table>

All values for room temperature.

**References**


**Thermal diffusivity, \( a \)**

- Materials with a high value of \( a \) rapidly adjust their temperature to that of surroundings, because they conduct heat rapidly in comparison to their volumetric heat capacity; do not require much energy to reach thermal equilibrium.
  
  e.g. Cu \( a = 112 \times 10^{-6} \text{ m}^2/\text{s} \)
  nylon \( a = 0.09 \times 10^{-6} \text{ m}^2/\text{s} \)
  wood \( a = 0.082 \times 10^{-6} \text{ m}^2/\text{s} \)

**Thermal conductivity of a foam, \( \lambda^* \).**

\( \lambda^* \) — contributions from
- conduction through solid, \( \lambda_s^* \)
- conduction through gas, \( \lambda_g^* \)
- convection within cells, \( \lambda_c^* \)
- radiation through cell walls and across voids, \( \lambda_r^* \)

\[ \lambda^* = \lambda_s^* + \lambda_g^* + \lambda_c^* + \lambda_r^* \]

- Conduction through solid: \( \lambda_s^* = \eta \lambda_s (\rho^*/\rho_s) \) \( \eta \) = efficiency factor \( \sim 2/3 \)
- Conduction through gas: \( \lambda_g^* = \lambda_g (1 - \rho^*/\rho_s) \)
For example, 2.5% dense closed-cell polystyrene foam:

\[
\lambda^* = 0.040 \text{ W/mK; } \lambda_s^* = 0.15 \text{ W/mK; } \lambda_g^* = 0.025 \text{ W/mK (air)}
\]

\[
\lambda_s^* + \lambda_g^* = \frac{2}{3} (0.15)(0.025) + (0.025)(0.975)
= 0.003 + 0.024
= 0.027 \text{ W/mK}
\]

- Most of conductivity comes from conduction through gas
- Foams for isolation blown with low \(\lambda_g\) gases
- Problem with aging — low \(\lambda_g\) gases diffuse out of foam over time, air diffuses in; \(\lambda_g^* \uparrow\)

**Convection within the cell**

- Gas rises and falls due to density changes with temperature
- Density changes — buoyancy forces
- Also have viscous forces from drag of gas as it moves past cell wall

Convection is important when Rayleigh number \(> 1000\)

\[
R_a = \frac{\rho g \beta \Delta T_c l^3}{\mu a}
\]

- \(\rho = \) density of gas
- \(g = \) grav. acceleration
- \(\beta = \) volume expansion
- \(\Delta T_c = \) temp. diff. across the cell
- \(l = \) cell size
- \(\mu = \) dynamic viscosity of gas
- \(a = \) thermal diffusion

\(\text{for a gas} = 1/T \) (isobaric)
Convection

For $R_a = 1000$ air $p = p_{atm}$ $T =$ room temp $\beta = 1/T = 1/300 \, (^\circ K^{-1})$.

\[ \Delta T_c = 1^\circ K \] $\mu_{air} = 2 \times 10^{-5} \text{ Pa} \cdot \text{s}$ $\rho_{air} = 1.2 \text{ kg/m}^3$

\[ a_{air} = 2.0 \times 10^{-5} \text{ m}^2/\text{s} \]

\[ \Rightarrow l = 20 \text{ mm} \]

- Convection important if cell size $> 20 \text{ mm}$
- Most foams: cell size $< 1 \text{ mm} \Rightarrow$ convection negligible

Radiation

- Hect flux passing by radiation, $q_r^0$, from surface at temperature $T_1$, to one at a lower temperature $T_0$, with a vacuum between them, is:

\[ q_r^0 = \beta_1 \sigma (T_1^4 - T_0^4) \quad \text{Stefan’s law} \]

\[ \sigma = \text{Stefan’s constant} = 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4 \]

\[ \beta_1 = \text{constant} (< 1) \text{ describing emissivity of the surfaces} \]

(emitted radiant flux per unit area of sample relative to black body radiator at same temperature and conditions; black body absorbs all energy; black body emissivity =1)
Radiation

- If put foam between two surfaces, heat flux is reduced, since radiation is absorbed by the solid and reflected by cell walls

- Attenuation $q_r = q_r^0 \exp(-K^*t^*)$  
  Beer’s law
  $K^*$ = extinction coefficient for foam
  $t^*$ = thickness of foam

- For optically thin walls and struts ($t < 10\mu m$) (transparent to radiation)
  
  $K^* = (\rho^*/\rho_s)K_s$

- Heat flux by radiation then:
  
  $q_r = \lambda^*_r \frac{dT}{dx}$

  $q_r = \beta_1 \sigma(T_1^4 - T_0^4) \exp[-(\rho^*/\rho_s)K_s t^*] = \lambda^*_r \frac{dT}{dx}$

- Obtain $\lambda^*_r$ using some approximations
Approximations:

\[
\frac{dT}{dx} \approx \frac{T_1 - T_0}{t^*} = \frac{\Delta T}{t^*}
\]

\[
T_1^4 - T_0^4 \approx 4 \Delta T \bar{T}^3 \quad \bar{T} = \left( \frac{T_1 - T_0}{2} \right)
\]

\[q_r = \beta_1 \sigma 4 \Delta T \bar{T}^3 \exp \left[ -\left( \rho^* / \rho_s \right) K_s t^* \right] = \lambda_r^* \frac{\Delta T}{dx}
\]

\[\lambda_r^* = 4\beta_1 \sigma \bar{T}^3 t^* \exp \left[ -\left( \rho^* / \rho_s \right) K_s t^* \right]
\]

\[
as \rho^*/\rho_s \downarrow \quad \lambda_r^* \uparrow
\]

**Thermal conductivity**

- Relative contributions of \(\lambda_s^*, \lambda_g^*, \lambda_r^*\) shown in Fig. 7.1
  - largest contribution \(\lambda_g^*\)
- \(\lambda^*\) plotted against relative density Fig. 7.2
  - minimum between \(\rho^*/\rho_s\) of 0.03 and 0.07
  - at which point \(\lambda^*\) only slightly larger than \(\lambda_s^*\)
  - at low \(\rho^*/\rho_s\), \(\lambda^*\) increases - increasing transparency to radiation (also, walls may rupture)
  - tradeoff: as \(\rho^*/\rho_s\) goes down, \(\lambda_s^*\) goes down, but \(\lambda_r^*\) goes up
Thermal Conductivity

Cond. Vs. Relative Density

Cond. vs. Cell Size

\( \lambda^* \) plotted against cell size Fig. 7.3

- \( \lambda^* \) increases with cell size
- Radiation reflected less often

Note: aerogels

- Pore size < 100nm
- Mean free path of air at ambient pressure = 68 nm
  \( \rightarrow \) average distance molecules move before collision with another molecule
- Aerogels — pore size < mean free path of air — reduced conduction through gas

**Specific heat** \( C_p \)

- Specific heat — energy required to raise temperature of unit mass by unit temperature

\[ C_p^* = C_{ps} \quad [\text{J/kg} \cdot \text{K}] \]

**Thermal expansion coefficient**

\[ \alpha^* = \alpha_s \quad (\text{consider foam as framework}) \]

(but if closed-cell foam cooled dramatically — gas can freeze, collapsing the cells; or if heated — gas expands, increasing the internal pressure and strains)
Thermal shock resistance

- If material subjected to sudden change in surface temperature - induces thermal stresses at surface, plus cracking and spalling

- Consider material at $T_1$ dropped into water at $T_2$ ($T_1 > T_2$)
  - Surface temperature drops to $T_2$, contracting surface layers
  - Thermal strain $\epsilon_T = \alpha \Delta T$

- If surface bonded to underlying block of material - constrained to original dimensions

\[
\sigma = \frac{E \alpha \Delta T}{1 - \nu} \quad \text{in the surface}
\]

- Cracking/spalling when $\sigma = \sigma_f$

\[
\Delta T = \sigma_f \frac{1 - \nu}{E \alpha} = \text{critical } \Delta T \text{ to just cause cracking}
\]

- For foam: (open cells)

\[
\Delta T^*_c = \frac{0.2 \sigma_{fs}(\rho^*/\rho_s)^{3/2}(1 - \nu^*)}{E_s (\rho^*/\rho_s)^2 \alpha_s} = \frac{0.2 \sigma_{fs}(1 - \nu)}{E_s \alpha_s} = \frac{0.2}{(\rho^*/\rho_s)^{1/2}} \Delta T_{cs}
\]

- As foam density goes down, $\Delta T^*_c$ goes up

  firebrick - porous ceramic
Case study: optimization of foam density for thermal insulation

- There is an optimal foam density for a given thermal insulation problem
- Already saw $\lambda^*$ has a minimum as a $f(\rho^*/\rho_s)$
- Typically, have a constraint on the foam thickness, $t^*$, $t^* =$constant
  \[
  \lambda^* = \frac{2}{3} (\rho^*/\rho_s) \lambda_s + (1 - \rho^*/\rho_s) \lambda_g^* + 4\beta_1 \sigma \bar{T}^3 t^* \exp[-K_s (\rho^*/\rho_s) t^*]
  \]
- What is optimum $\rho^*/\rho_s$ for a given $t^*$?
  \[
  \frac{d\lambda^*}{d(\rho^*/\rho_s)} = 0 \Rightarrow (\rho^*/\rho_s)_{opt} = \frac{1}{K_s t^*} \ln \left[ \frac{4K_s \beta_1 \sigma \bar{T}^3 t^*}{\frac{2}{3} \lambda_s - \lambda_g} \right]
  \]
- As given thickness $t^*$ increases, $(\rho^*/\rho_s)_{opt}$ decreases
- As $\bar{T}$ increases, $(\rho^*/\rho_s)_{opt}$ increases

  e.g. coffee cup $t^* = 3\text{mm}$ $(\rho^*/\rho_s)_{opt} = 0.08$
  refrigerator $t^* = 50\text{mm}$ $(\rho^*/\rho_s)_{opt} = 0.02$

(see PP slide Table 7.3 for data used in calculations)
Case Study: Optimization of Relative Density

Case Study: Optimum Relative Density

<table>
<thead>
<tr>
<th>Table 7.3</th>
<th>Data for optimization case study</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extinction coefficient of solid polymer, $K_s$</td>
<td>$5.67 \times 10^4 \text{ m}^{-1}$</td>
</tr>
<tr>
<td>Emissivity factor, $\beta_1$</td>
<td>0.5</td>
</tr>
<tr>
<td>Conductivity of solid polymer, $\lambda_s$</td>
<td>$0.22 \text{ W/m K}$</td>
</tr>
<tr>
<td>Conductivity of gas, $\lambda_g$</td>
<td>$0.02 \text{ W/m K}$</td>
</tr>
<tr>
<td>Mean temperature, $T$</td>
<td>$300^\circ \text{K}$</td>
</tr>
<tr>
<td>Stefan’s constant, $\sigma$</td>
<td>$5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$</td>
</tr>
</tbody>
</table>

Case study: insulation for refrigerators

- Insulation reduces energy cost, but has a cost itself
- Total cost is the cost of insulation plus the cost of energy lost by heat transfer through walls
- Objective function: minimize total cost
- given: \( x = \) thickness of insulation \( C_M = \) cost of insulation/mass
  \( \Delta T = \) temp. diff. across insulation \( C_E = \) cost of energy / joule
  \( t_l = \) design life of refrigerator \( C_T = \) total cost/area

\[
C_T = x \rho^* C_M + \lambda \frac{\Delta T}{x} t_l C_E
\]

(heat flux \( q = \lambda \frac{\Delta T}{x} \frac{J}{m^2 s} \))

Define:
\[
M_1 = \frac{1}{\rho^* C_M} \quad M_2 = \frac{1}{\lambda}
\]

\[
\frac{C_T}{x} = \frac{1}{M_1} + \left[ \frac{\Delta T}{x^2 t_l C_E} \right] \frac{1}{M_2}
\]
• The terms are equal when:

\[ M_2 = \left[ \frac{\Delta T}{x^2} t_l C_E \right] M_1 \]

coupling constant

• Family of parallel straight lines of constant value \( \frac{\Delta T}{x^2} t_l C_E \)

• Fig. 13.11 \( \Delta T = 20^\circ \) \( x = 10\text{mm} \) \( C_E = 0.01/\mu\text{J} \)

Two lines for \( t_2 = 10 \) years and \( t_l = 1 \) month
(note error in book \( t_l = 10 \) years line should be moved over)

• Also plotted a set of curved contours - plots of \( C_T/x \):
  
  ○ As move up and to the right of plot, the value of \( C_T/x \) decreases

• For \( t_l = 10 \) years \( \Rightarrow \) phenolic foam \( \rho^* = 0.035 \text{Mg/m}^3 \)
  
  For \( t_l = 1 \) month \( \Rightarrow \) EPS
  
  PP \( \rho^* = 0.02 \text{Mg/m}^3 \)
Case Study:
Insulation for Refrigerators
