1. Write each of the following functions (of $t$) in the form $A \cos(\omega t - \phi)$. In each case, begin by drawing a right triangle with sides $a$ and $b$.

(a) $\cos(2t) + \sin(2t)$.

(b) $\cos(\pi t) - \sqrt{3} \sin(\pi t)$.

(c) $\text{Re} \left( e^{it(\sqrt{2} + 2i)} \right)$.

Recall the geometric derivation of the general sinusoid formula

$$a \cos(\omega t) + b \sin(\omega t) = \langle a, b \rangle \cdot (\cos(\omega t), \sin(\omega t)) = \sqrt{a^2 + b^2} \cos(\omega t - \phi),$$

where $\phi$ is the angle of the first vector. This is where the triangle comes in - we will draw the vector of coefficients to determine its magnitude and direction. (Equivalently, we are determining the polar coordinates of the complex number $a + bi$.)

(a) $\cos(2t) + \sin(2t)$: $(a, b) = (1, 1)$.

Here, the right triangle has hypotenuse $\sqrt{1 + 1} = \sqrt{2}$, so $A = \sqrt{2}$. Both summands have angular frequency 2, so $\omega = 2$. $\phi$ is the angle of the triangle, which is $\pi/4$, so $\cos(2t) + \sin(2t) = \sqrt{2} \cos(2t - \pi/4)$.

(b) $\cos(\pi t) - \sqrt{3} \sin(\pi t)$: $(a, b) = (1, -\sqrt{3})$. 
This right triangle has hypotenuse $\sqrt{1^2 + (-\sqrt{3})^2} = 2$ and angle $-\pi/3$. So $\cos(\pi t) - \sqrt{3} \sin(\pi t) = 2 \cos(\pi t - (-\pi/3)) = 2 \cos(\pi t + \pi/3)$.

(c) $e^{it} = \cos(t) + i \sin(t)$ and $\frac{1}{2 + 2i} = \frac{1}{2 + 2i} \cdot \frac{2 - 2i}{2 - 2i} = \frac{2 - 2i}{2^2 + 2^2} = \frac{1 - i}{4}$. Multiply out and take the real part of the product to obtain $\text{Re} \left( \frac{e^{it}}{2 + 2i} \right) = \frac{1}{4} \cos(t) + \frac{1}{4} \sin(t)$.

For $\frac{1}{4} \cos(t) + \frac{1}{4} \sin(t)$, $(a, b) = (\frac{1}{4}, \frac{1}{4})$, which gives the same triangle as in (a), except scaled by $1/4$.

So $\text{Re} \left( \frac{e^{it}}{2 + 2i} \right) = \frac{1}{4} \cos(t) + \frac{1}{4} \sin(t) = \frac{\sqrt{2}}{4} \cos(t - \pi/4)$. 