18.085/18.0851 Computational Science and Engineering I **Summer 2020**

Homework 6

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6.1 Complex Exponentials of Fourier Series

Find the Fourier series representation using $e^{\pm inx}$ of the following function

$$f(x) = x, -\pi \leq x \leq \pi \tag{6.1}$$

6.2 Fourier Series over Half Intervals

Suppose we want to represent $f(x)$ defined over the half interval $[0, L]$, using the basis function $\sin \frac{n\pi}{L} x$ and $\cos \frac{n\pi}{L} x$. Since these basis functions are for functions over $[-L, L]$, we must extend f over to $[-L, 0]$ in order to use these functions. By "extend" we mean to define another function $g(x)$ such that $g(x) = f(x)$. Amid many possible extensions, we define the odd extension as

$$g(x) = \begin{cases} f(x), x \in [0, L] \\ -f(-x), x \in [-L, 0] \end{cases} \tag{6.2}$$

In other words, in the odd extension, g is odd. On the other, we can define the even extension as

$$g(x) = \begin{cases} f(x), x \in [0, L] \\ f(x), x \in [-L, 0] \end{cases} \tag{6.3}$$

In other words, in the even extension, g is even.

Now we let $f(x) = \cos(x)$ over $[0, L]$. Here are three tasks

- Draw the odd extension of f and calculate the Fourier series (note since the extension is odd, all cosine terms of the Fourier series are gone)
- Draw the even extension of f and calculate the Fourier series (note since the extension is even, all sine terms of the Fourier series are gone)
- With the knowledge acquired so far, re-read the Week 7 Lecture and understand what happened in equation (7.30). Feel free to discuss with me during office hour if you don't get it.

Hint: if you are stuck with the integrals, try the following trig identity

$$\cos(a)\cos(b) = \frac{1}{2}(\cos(a-b) + \cos(a+b)) \tag{6.4}$$

$$\cos(a)\sin(b) = \frac{1}{2}(\sin(b+a) + \sin(b-a)) \tag{6.5}$$

6.3 Laplace's Equation in the Cartesian Coordinate

Solve $\Delta u = 0$ INSIDE a square defined by $x = 0$, $x = 1$, $y = 0$, and $y = 1$, subject to the following boundary condition

$$\begin{cases} u(x,0) = 0 \\ u(x,1) = 0 \\ u(0,y) = 0 \\ u(1,y) = y \end{cases} \tag{6.6}$$

6.4 Laplace's Equation in the Polar Coordinate

Solve $\Delta u = 0$ INSIDE a circle of radius 1 and boundary condition $u(\theta) = \cos(10\theta)$