Modulation
Communications Systems

Signals are not always well matched to the media through which we wish to transmit them.

<table>
<thead>
<tr>
<th>signal</th>
<th>applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>audio</td>
<td>telephone, radio, phonograph, CD, cell phone, MP3</td>
</tr>
<tr>
<td>video</td>
<td>television, cinema, HDTV, DVD</td>
</tr>
<tr>
<td>internet</td>
<td>coax, twisted pair, cable TV, DSL, optical fiber, E/M</td>
</tr>
</tbody>
</table>

**Modulation** can improve match based on **frequency**.
Amplitude modulation can be used to match audio frequencies to radio frequencies. It allows parallel transmission of multiple channels.
Edwin Howard Armstrong invented the superheterodyne receiver, which made broadcast AM practical.

Edwin Howard Armstrong also invented and patented the “regenerative” (positive feedback) circuit for amplifying radio signals (while he was a junior at Columbia University). He also invented wide-band FM.
Amplitude, Phase, and Frequency Modulation

There are many ways to embed a “message” in a carrier.

Amplitude Modulation (AM) + carrier:  
\[ y_1(t) = (x(t) + C) \cos(\omega_c t) \]

Phase Modulation (PM):  
\[ y_2(t) = \cos(\omega_c t + kx(t)) \]

Frequency Modulation (FM):  
\[ y_3(t) = \cos \left( \omega_c t + k \int_{-\infty}^{t} x(\tau) d\tau \right) \]

**PM:** signal modulates instantaneous phase of the carrier.
\[ y_2(t) = \cos(\omega_c t + kx(t)) \]

**FM:** signal modulates instantaneous frequency of carrier.
\[ y_3(t) = \cos \left( \omega_c t + k \int_{-\infty}^{t} x(\tau) d\tau \right) \]
\[ \omega_i(t) = \omega_c + \frac{d}{dt} \phi(t) = \omega_c + kx(t) \]
Frequency Modulation

Compare AM to FM for $x(t) = \cos(\omega_m t)$.

AM: $y_1(t) = x(t) + C \cos(\omega_c t) = (\cos(\omega_m t) + 1.1) \cos(\omega_c t)$

FM: $y_3(t) = \cos \omega_c t + k \int_{-\infty}^{t} x(\tau) d\tau = \cos(\omega_c t + \frac{k}{\omega_m} \sin(\omega_m t))$

Advantages of FM:
- constant power
- no need to transmit carrier (unless DC important)
- bandwidth?
Frequency Modulation

Early investigators thought that narrowband FM could have arbitrarily narrow bandwidth, allowing more channels than AM.

\[
y_3(t) = \cos\left(\omega_c t + k \int_{-\infty}^{t} x(\tau) d\tau\right)
\]

\[
\phi(t)
\]

\[
\omega_i(t) = \omega_c + \frac{d}{dt} \phi(t) = \omega_c + k x(t)
\]

Small \( k \rightarrow \) small bandwidth. Right?
Frequency Modulation

Early investigators thought that narrowband FM could have arbitrarily narrow bandwidth, allowing more channels than AM. **Wrong!**

\[
y_3(t) = \cos\left(\omega_c t + k \int_{-\infty}^{t} x(\tau)d\tau\right)
\]

\[
= \cos(\omega_c t) \times \cos\left(k \int_{-\infty}^{t} x(\tau)d\tau\right) - \sin(\omega_c t) \times \sin\left(k \int_{-\infty}^{t} x(\tau)d\tau\right)
\]

If \( k \to 0 \) then

\[
\cos\left(k \int_{-\infty}^{t} x(\tau)d\tau\right) \to 1
\]

\[
\sin\left(k \int_{-\infty}^{t} x(\tau)d\tau\right) \to k \int_{-\infty}^{t} x(\tau)d\tau
\]

\[
y_3(t) \approx \cos(\omega_c t) - \sin(\omega_c t) \times \left(k \int_{-\infty}^{t} x(\tau)d\tau\right)
\]

Bandwidth of narrowband FM is the same as that of AM! (integration does not change the highest frequency in the signal)
Phase/Frequency Modulation

Find the Fourier transform of a PM/FM signal.

\[ y(t) = \cos(\omega_c t + mx(t)) = \cos(\omega_c t + m \sin(\omega_m t)) \]
\[ = \cos(\omega_c t) \cos(m \sin(\omega_m t)) - \sin(\omega_c t) \sin(m \sin(\omega_m t)) \]

\( x(t) \) is periodic in \( T = \frac{2\pi}{\omega_m} \), therefore \( \cos(m \sin(\omega_m t)) \) is periodic in \( T \).
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Phase/Frequency Modulation

Fourier transform of first part.

\[ x(t) = \sin(\omega_m t) \]
\[ y(t) = \cos(\omega_c t + m x(t)) = \cos(\omega_c t + m \sin(\omega_m t)) \]
\[ = \cos(\omega_c t) \cos(m \sin(\omega_m t))) - \sin(\omega_c t) \sin(m \sin(\omega_m t))) \]
\[ y_a(t) \]
\[ |Y_a(j\omega)| \]
\[ m = 50 \]

\[ \omega_c \]
\[ 100\omega_m \]
Phase/Frequency Modulation

Find the Fourier transform of a PM/FM signal.

\[ y(t) = \cos(\omega_c t + mx(t)) = \cos(\omega_c t + m \sin(\omega_m t)) \]

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\( x(t) \) is periodic in \( T = \frac{2\pi}{\omega_m} \), therefore \( \sin(m \sin(\omega_m t)) \) is periodic in \( T \).
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\( x(t) \) is periodic in \( T = \frac{2\pi}{\omega_m} \), therefore \( \sin(m \sin(\omega_m t)) \) is periodic in \( T \).
Phase/Frequency Modulation

Fourier transform of second part.

\[ x(t) = \sin(\omega_m t) \]
\[ y(t) = \cos(\omega_c t + mx(t)) = \cos(\omega_c t + m \sin(\omega_m t)) \]
\[ = \cos(\omega_c t) \cos(m \sin(\omega_m t)) - \sin(\omega_c t) \sin(m \sin(\omega_m t)) \]
\[ y_b(t) \]
\[ |Y_b(j\omega)| \quad m = 50 \]
Phase/Frequency Modulation

Fourier transform.

\[ x(t) = \sin(\omega_m t) \]
\[ y(t) = \cos(\omega_c t + mx(t)) = \cos(\omega_c t + m \sin(\omega_m t)) \]
\[ = \frac{\cos(\omega_c t) \cos(m \sin(\omega_m t))}{y_a(t)} - \frac{\sin(\omega_c t) \sin(m \sin(\omega_m t))}{y_b(t)} \]

\[ |Y(j\omega)| \quad m = 50 \]
Frequency Modulation

Wideband FM is useful because it is robust to noise.

**AM:** \[ y_1(t) = (\cos(\omega_m t) + 1.1) \cos(\omega_c t) \]

**FM:** \[ y_3(t) = \cos(\omega_c t + m \sin(\omega_m t)) \]

FM generates a redundant signal that is resilient to additive noise.
Summary

Modulation is useful for matching signals to media.

Examples: commercial radio (AM and FM)

Close with unconventional application of modulation – in microscopy.
6.003 Model of a Microscope

Microscope = low-pass filter

Courtesy of Stanley Hong, Jekwan Ryu, Michael Mermelstein, and Berthold K. P. Horn. Used with permission.
Phase-Modulated Microscopy

Courtesy of Stanley Hong, Jekwan Ryu, Michael Mermelstein, and Berthold K. P. Horn. Used with permission.
Phase-Modulated Microscopy

Poster: \( \cos(\omega_c y + f(x,y)) \)

Projector: \( \cos(\omega_c y) \)

Poster with Projector: \( \cos(\omega_c y) \cos(\omega_c y + f(x,y)) \)

Modulated illumination enables low-pass system (eyes) to detect high spatial frequencies

Courtesy of Stanley Hong, Jekwan Ryu, Michael Mermelstein, and Berthold K. P. Horn. Used with permission.
Modulated illumination enables low-pass system (eyes) to detect high spatial frequencies

Courtesy of Stanley Hong, Jekwan Ryu, Michael Mermelstein, and Berthold K. P. Horn. Used with permission.
Images are 2 dimensional
→ need 2D Fourier Transform
many frequencies + many orientations = many images

low resolution

high resolution

Courtesy of Stanley Hong, Jekwan Ryu, Michael Mermelstein, and Berthold K. P. Horn. Used with permission.
Standing-wave illumination spectrum

Thanks to M. Mermelstein

Courtesy of Stanley Hong, Jekwan Ryu, Michael Mermelstein, and Berthold K. P. Horn. Used with permission.
many frequencies + many orientations = many images

courtesy of Stanley Hong, Jekwan Ryu, Michael Mermelstein, and Berthold K. P. Horn. Used with permission.
Courtesy of Stanley Hong, Jekwan Ryu, Michael Mermelstein, and Berthold K. P. Horn. Used with permission.
Twinkling decoded into sub-pixel image

Courtesy of Stanley Hong, Jekwan Ryu, Michael Mermelstein, and Berthold K. P. Horn. Used with permission.
Uniform Illumination

Structured Illumination

Courtesy of Stanley Hong, Jekwan Ryu, Michael Mermelstein, and Berthold K. P. Horn. Used with permission.
Measurement of PSF

Courtesy of Stanley Hong, Jekwan Ryu, Michael Mermelstein, and Berthold K. P. Horn. Used with permission.
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Measurement of PSF

Measured diameter = 290 nm
Predicted diameter = 250 nm
Diameter lens alone = 1,500 nm

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