Class 18: Outline

Hour 1:
  Levitation
  Experiment 8: Magnetic Forces

Hour 2:
  Ampere’s Law
Review: Right Hand Rules

1. Torque: Thumb = torque, fingers show rotation
2. Feel: Thumb = I, Fingers = B, Palm = F
3. Create: Thumb = I, Fingers (curl) = B
Last Time: Dipoles
Magnetic Dipole Moments

\[ \vec{\mu} = IA \hat{n} = I \vec{A} \]

**Generate:**

**Feel:** \[ U_{Dipole} = -\vec{\mu} \cdot \vec{B} \]

1) Torque to align with external field
2) Forces as for bar magnets (seek field)
Some Fun: Magnetic Levitation
Put a Frog in a 16 T Magnet…

For details: http://www.hfml.sci.kun.nl/levitate.html
How does that work?
First a BRIEF intro to magnetic materials
Para/Ferromagnetism

Applied external field $B_0$ tends to align the atomic magnetic moments (unpaired electrons)
Diamagnetism


If no magnetic moments (unpaired electrons) then this effect dominates.
Back to Levitation
Levitating a Diamagnet

1) Create a strong field (with a field gradient!)
2) Looks like a dipole field
3) Toss in a frog (diamagnet)
4) Looks like a bar magnet pointing opposite the field
5) Seeks lower field (force up) which balances gravity

Most importantly, its stable:
Restoring force always towards the center
Using $\nabla B$ to Levitate

- Frog
- Strawberry
- Water Droplets
- Tomatoes
- Crickets

For details: [http://www.hfml.ru.nl/levitation-movies.html](http://www.hfml.ru.nl/levitation-movies.html)
Demonstrating: Levitating Magnet over Superconductor
Perfect Diamagnetism: “Magnetic Mirrors”
Perfect Diamagnetism: “Magnetic Mirrors”

No matter what the angle, it floats -- STABILITY
Using $\nabla B$ to Levitate

A Sumo Wrestler

For details: http://www.hfml.sci.kun.nl/levitate.html
Two PRS Questions Related to Experiment 8: Magnetic Forces
Experiment 8: Magnetic Forces (Calculating $\mu_0$)

Experiment Summary:
Currents feel fields
Currents also create fields

Recall… Biot-Savart
The Biot-Savart Law

Current element of length $ds$ carrying current $I$ produces a magnetic field:

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I d\mathbf{s} \times \hat{r}}{r^2}$$
Today:  
3rd Maxwell Equation:  
Ampere’s Law  

Analogous (in use) to Gauss’s Law
Gauss’s Law – The Idea

The total “flux” of field lines penetrating any of these surfaces is the same and depends only on the amount of charge inside.
Ampere’s Law: The Idea

In order to have a B field around a loop, there must be current punching through the loop.
Ampere’s Law: The Equation

\[ \oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc} \]

The line integral is around any closed contour bounding an open surface \( S \).

\( I_{enc} \) is current through \( S \):

\[ I_{enc} = \int_S \vec{J} \cdot d\vec{A} \]
PRS Question: Ampere’s Law
# Biot-Savart vs. Ampere

| Biot-Savart Law | $\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2}$ | general current source  
ex: finite wire  
wire loop |
|-----------------|-------------------------------------------------|----------------------------------|
| Ampere’s law    | $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$ | symmetric current source  
ex: infinite wire  
infinite current sheet |
Applying Ampere’s Law

1. Identify regions in which to calculate B field
   Get B direction by right hand rule
2. Choose Amperian Loops S: Symmetry
   B is 0 or constant on the loop!
3. Calculate $\int \mathbf{B} \cdot d\mathbf{s}$
4. Calculate current enclosed by loop S
5. Apply Ampere’s Law to solve for B

$$\int \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{enc}$$
Always True, Occasionally Useful

Like Gauss’s Law,

Ampere’s Law is **always** true

However, it is only useful for calculation in certain specific situations, involving highly symmetric currents.

Here are examples…”
Example: Infinite Wire

A cylindrical conductor has radius $R$ and a uniform current density with total current $I$

Find $B$ everywhere

Two regions:
(1) outside wire ($r \geq R$)
(2) inside wire ($r < R$)
Ampere’s Law Example: Infinite Wire

Amperian Loop:
B is Constant & Parallel
I Penetrates
Example: Wire of Radius $R$

Region 1: Outside wire ($r \geq R$)

Cylindrical symmetry $\rightarrow$

Amperian Circle

B-field counterclockwise

\[
\oint \vec{B} \cdot d\vec{s} = B \oint d\vec{s} = B \left( 2\pi r \right) = \mu_0 I_{enc} = \mu_0 I
\]

\[
\vec{B} = \frac{\mu_0 I}{2\pi r} \text{ counterclockwise}
\]
Example: Wire of Radius $R$

Region 2: Inside wire ($r < R$)

$$\oint \vec{B} \cdot d\vec{s} = B \oint d\vec{s} = B \left( 2\pi r \right)$$

$$\quad = \mu_0 I_{enc} = \mu_0 I \left( \frac{\pi r^2}{\pi R^2} \right)$$

$$\vec{B} = \frac{\mu_0 Ir}{2\pi R^2} \text{ counterclockwise}$$

Could also say: $J = \frac{I}{A} = \frac{I}{\pi R^2}$; $I_{enc} = JA_{enc} = \frac{I}{\pi R^2} \left( \pi r^2 \right)$
Example: Wire of Radius $R$

$$B_{in} = \frac{\mu_0 Ir}{2\pi R^2}$$

$$B_{out} = \frac{\mu_0 I}{2\pi r}$$
A cylindrical conductor has radius R and a non-uniform current density with total current:

$$\vec{J} = J_0 \frac{R}{r}$$

Find B everywhere
Applying Ampere’s Law

In Choosing Amperian Loop:
• Study & Follow Symmetry
• Determine Field Directions First
• Think About Where Field is Zero
• Loop Must
  • Be Parallel to (Constant) Desired Field
  • Be Perpendicular to Unknown Fields
  • Or Be Located in Zero Field
Other Geometries
Helmholtz Coil
Closer than Helmholtz Coil
Multiple Wire Loops
Multiple Wire Loops – Solenoid
Magnetic Field of Solenoid

loosely wound  tightly wound

For ideal solenoid, B is uniform inside & zero outside
Magnetic Field of Ideal Solenoid

Using Ampere’s law: Think!

\[
\begin{align*}
\oint \mathbf{B} \cdot d \mathbf{s} &= \int_1 \mathbf{B} \cdot d \mathbf{s} + \int_2 \mathbf{B} \cdot d \mathbf{s} + \int_3 \mathbf{B} \cdot d \mathbf{s} + \int_4 \mathbf{B} \cdot d \mathbf{s} \\
&= B l + 0 + 0 + 0 + 0
\end{align*}
\]

\[I_{enc} = n l I \quad n: \text{ turn density}\]

\[\oint \mathbf{B} \cdot d \mathbf{s} = B l = \mu_0 n l I\]

\[n = N / L : \# \text{ turns/unit length}\]

\[B = \frac{\mu_0 n l I}{l} = \mu_0 n I\]
Demonstration: Long Solenoid
A sheet of current (infinite in the y & z directions, of thickness 2d in the x direction) carries a uniform current density:

\[
\vec{J}_s = J \hat{k}
\]

Find B everywhere
Ampere’s Law: Infinite Current Sheet

Amperian Loops:

B is Constant & Parallel OR Perpendicular OR Zero OR I Penetrates
Solenoid is Two Current Sheets

Field outside current sheet should be half of solenoid, with the substitution:

$$nI = 2dJ$$

This is current per unit length (equivalent of $\lambda$, but we don’t have a symbol for it)
Ampere’s Law: \[ \int \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{enc} \]

Long Circular Symmetry

(Infinite) Current Sheet

Solenoid = 2 Current Sheets

Torus
Brief Review Thus Far…
Maxwell’s Equations (So Far)

Gauss's Law: \[ \oiint_{S} \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{in}}}{\varepsilon_0} \]

Electric charges make diverging Electric Fields

Magnetic Gauss's Law: \[ \oiint_{S} \mathbf{B} \cdot d\mathbf{A} = 0 \]

No Magnetic Monopoles! (No diverging B Fields)

Ampere's Law: \[ \oint_{C} \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{enc}} \]

Currents make curling Magnetic Fields