Forecasting and Inventory Management of Short Life-Cycle Products

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Short Life-Cycle Products

(Screenshot of the home page from Dell Inc.: http://www.dell.com – last accessed June 29, 2004.)
Short Life-Cycles

- **Causes**
  - Fast Changing Consumer Preferences
  - Rapid Rate of Innovation

- **Procurement Issues**
  - Forecasting with no historical data
  - Long lead-times
  - Perishable Inventory

- **Introduction Time**
Outline

- Forecasting new product introduction
- Procurement issues
  - Model Formulation
  - Optimal Control Solution
  - Discussion
- Case Study
New Product Introduction

- No past sales data
  - Time-series useless
- But multiple-product environment
  - Some level of predictability
  - Independence (serve different needs)
Diffusion Theory

- “[Innovation] is communicated through certain channels over time among members of a social system”

- Marketing application:
  - Mass Media
  - Word of Mouth
The Bass Model

Noncumulative adoption vs. time

- Internal Influence
- External Influence
The Bass Model: Assumptions

- Market potential remains consistent over time
- Independence of other innovations
- Product and Market characteristics do not influence diffusion patterns

- Competition?
The Bass Model: Sales Evolution

\[
\frac{dN_t}{dt} = p + q \frac{N_t}{m} (m - N_t) \alpha_t
\]

Current Sales

Mass Media

Word-of-Mouth

Remaining market potential

Seasonality Coeff.

Many applications at Eastman Kodak, IBM, Sears, AT&T…
Case-Study: PC Manufacturer

- Monopolist (strongly differentiated)
- Life-cycle: 1-2 years
- Peak sales timing is predictable $T^*$
  - Christmas peak
- Typical seasonal variation in demand $\alpha_t$
  - End-of-quarter effect
- Information on total life-cycle sales $m$
Numerical Example (M2)

Sales Evolution of High-End Computers (M2)

Parameters Estimation

- Estimation of $p, q, m, \alpha_t$
- Nonlinear Least Squares
- R-squared above .9
Numerical Example (M2)

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Procurement Issues

- Need to place orders in advance
  - Long lead-times
  - Cost advantage, timely delivery
- Inventory/Backorder costs
- Schedule the procurement to meet the (random) demand, evolving according to Bass’ Model
Model Description

- State: Cumulative Procurement $V_t$
- Control: Instantaneous Procurement $u_t$
- Transition function
  $$V_t = u_t \quad V_0 = 0$$
- Finite-Horizon $T$ Optimization
- Discounted cost at rate $r$
Cost Parameters

- Instantaneous trade-off between
  - Inventory holding costs $h(V_t - N_t)^+$
  - Backorder costs $p(N_t - V_t)^+$
    \[ P_t(V_t - N_t) \]

- Terminal trade-off between
  - Salvage inventory loss $l(V_T - N_T)^+$
  - Shortage costs $s(N_T - V_T)^+$
    \[ Q_T(N_T - V_T) \]
Optimal Control Model

\[
\min_u J = \int_0^T e^{-rt} \int_{N_t} P_t(V_t - N_t)\psi(N_t)dN_tdt + e^{-rT} \int_{N_T} Q_T(V_T - N_T)\psi(N_T)dN_T
\]

such that: \( V_t = u_t \) and \( u_t \geq 0 \)

- Timely delivery of customer orders?
- What if we do not want to serve all the demand?
- Why no chance constraints instead?
Hamiltonian function

Define

$$\lambda_t = \nabla_v J^*(t, V_t^*)$$

Hamiltonian

$$H(V, u, \lambda) = P_t(V, u) + \lambda u$$
Pontryagin Minimum Principle

1. Adjoint Equation
   \[ \lambda_t = -\frac{\partial H(V^*_t, u^*_t, \lambda_t)}{\partial V_t} \]

2. Boundary Condition
   \[ \lambda_T = \frac{d}{dV_T} \left[ \int_{N_T} Q_T(N_T - V_T)\psi_T(N_T) dN_T \right] \]

3. Optimality of Control
   \[ u^*_t = \arg \min_{u \geq 0} H(V^*_t, u, \lambda_t) \]
Case I: \( \frac{b}{b+h} \leq \frac{s}{s+l} \)

- Maintain the same service level
  \[ \Psi_t(V_t) = \frac{b}{b+h} \]
- Impulse at the end of horizon
  \[ \Psi_T(V_T) = \frac{s}{s+l} \]
Procurement Policy

Cumulative units

Cumulative Inventory

Expected Cumulative Demand

Time
Case II: \[
\frac{b}{b+h} > \frac{s}{s+l}
\]

- For \(0 \leq t \leq \hat{t}\), keep the same service level

\[
\Psi_t(V_t) = \frac{b}{b+h}
\]

- For \(\hat{t} \leq t \leq T\),
  - Do not purchase anymore
  - Decrease gradually the service level down to \(\frac{s}{s+l}\)
Desired/Effective Service Level

- In practice, backorder costs are hard to evaluate...
- Instead, evaluate the desired SL \( \frac{b}{b+h} \)

- Terminal service level: switch the customers to an upgraded model
  - Loss of goodwill
  - Higher cost

- Case II is typical in practice
  - Terminal SL < Lifetime SL
Procurement Policy

Cumulative units $\hat{V}$

Expected Cumulative Demand

Cumulative Inventory

Phase-Out Period

Time $\hat{t}$
Revised Multiple-Period Implementation

- Update the estimation of $p, q, m$
Time-varying costs

- Time-varying costs
  - Decreasing purchase costs
  - 30% in less than 6 months

- Underage Costs
  - Backorder penalty $b$
  - Save the cost decrease $c_t$
  - Save from the cost of capital $-rc_t$
    - Decreasing over time
    - Hence, increasing service level
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PC Manufacturer: New Product Introduction

1. When should it be launched? May or August?
2. How much and when should we order?

Sensitivity Analysis on the Lifetime Service Level
Parameters Estimation

- Randomness summarized in $m, p, q$
- Estimation of the size of the market $m$
- Estimation of the peak time $T^*$
  - Relation between $p$ and $q$
- Past Product Introductions (M1-M4)
  - Estimation of the distribution of $q$
  - Sensitivity Analysis on variance
Demand Estimation (May)

Service Levels

- Lifetime service level
  - 95% vs. 99%?
- Terminal service level: 33%

- Hence, Case II, i.e.
  - Purchase period
  - Phase-out period
Procurement Decisions (May)

- Longer Phase-Out with low SL
- Reduce Procurement after peak season

Safety Stock Evolution

- Deplete SS in the last quarter
- Avg SS=5 or 8 weeks of demand

Additional Insights

- Launching the product early requires less inventories.

- With decreasing costs,
  - Reduced service levels (but increasing over time); hence, less inventory.
  - Delayed procurement cutoff time.
Conclusions

- Application-driven research
  - Adapt Bass’ Model
  - Optimal Control

- Additional issues:
  - Effectiveness of Bass’ Model?
  - Backorder costs vs. Service Level?
  - Terminal shortage penalty vs. Stopping time?
Thank you

Questions?