The node equations are:

\[ e_1 : \left( c_2 \frac{d}{dt} + 64 \right) e_1 - c_2 \frac{d}{dt} e_2 = 0 \]

\[ -c_2 \frac{d}{dt} e_1 + \left( c_1 \frac{d}{dt} + c_2 \frac{d}{dt} + G_5 \right) e_2 - c_1 \frac{d}{dt} e_3 = 0 \]

\[ -c_1 \frac{d}{dt} e_2 + \left( c_1 \frac{d}{dt} + G_3 \right) e_3 = 0 \]

Plugging in component values,

\[ \left( 2 \frac{d}{dt} + 1 \right) e_1 - 2 \frac{d}{dt} e_2 \]

\[ -2 \frac{d}{dt} e_1 + \left( 3 \frac{d}{dt} + 1 \right) e_2 - \frac{d}{dt} e_3 = 0 \]

\[ \frac{d}{dt} e_2 + \left( \frac{d}{dt} + 0.5 \right) e_3 = 0 \]

To find the solution, assume

\[ e_1(t) = E_1 e^{st} \]

\[ e_2(t) = E_2 e^{st} \]

Then

\[ (2s+1) E_1 - 2s E_2 = 0 \]

\[ -2s E_1 + (3s+1) E_2 - s E_3 = 0 \]

\[ -s E_2 + (s+0.5) E_3 = 0 \]
In matrix form,

\[
\begin{bmatrix}
2s+1 & -2s & 0 \\
-2s & 3s+1 & -s \\
0 & -s & s+0.5
\end{bmatrix}
\begin{bmatrix}
E_1 \\
E_2 \\
E_3
\end{bmatrix}
= 0
\]

\[= M(s) E\]

For there to be a nontrivial solution,

\[
\det(M(s)) = 0
\]

\[= s^2 + 3.5s + 0.5\]

This equation can be solved by using the quadratic formula, or a polynomial solver. The roots are:

\[s_1 = -0.2\sec^{-1}\]

\[s_2 = -0.5\sec^{-1}\]

Solve for \(E\) in each case:

\[s_1 = -0.2\]

\[\Rightarrow M(s) = \begin{bmatrix}
0.6 & 0.4 & 0 \\
+0.4 & 0.4 & +0.2 \\
0 & +0.2 & 0.3
\end{bmatrix}\]

Normally, would solve by row reduction. Because of the zeros in \(M\), can solve as follows: Set \(E_3 = 1\). From last row of \(M\),

\[+0.2E_2 + 0.3E_3 = 0\]

\[\Rightarrow E_2 = -1.5\]
From row 1 of $M$,

\[0.6 E_1 + 0.4 E_2 = 0\]

\[\Rightarrow E_1 = 0\]

So

\[E_1 = \begin{bmatrix} 1 \\ -1.5 \\ 0 \end{bmatrix}\]

(Of course, any multiple of this is also a solution.)

\[s_2 = -0.5 \Rightarrow M(s) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -0.5 & 0.5 \\ 0 & 0.5 & 0 \end{bmatrix}\]

From row 1 (or row 3),

\[+E_2 = 0 \Rightarrow E_2 = 0\]

Arbitrarily choose $E_3 = 1$. Then from row 2,

\[+E_1 - 0.5 E_2 + 0.5 E_3 = 0\]

\[\Rightarrow E_1 = -0.5\]

Therefore,

\[E_2 = \begin{bmatrix} -0.5 \\ 0 \\ 1 \end{bmatrix} \quad \text{(or any multiple)}\]

**Total solution**

The total solution is given by

\[\begin{pmatrix} e_1(t) \\ e_2(t) \\ e_3(t) \end{pmatrix} = a E_1 e^{s_1 t} + b E_2 e^{s_2 t}\]
From the circuit,
\[ U_1(t) = e_3(t) - e_2(t) \]
\[ U_2(t) = e_1(t) - e_2(t) \]

To match the initial conditions,
\[ U_1(0) = 10 \text{ V} = a(1+1.5)e^0 + b(1-0)e^0 \]
\[ = 2.5a + b \]
\[ U_2(0) = 0 \text{ V} = a(1+1.5)e^0 + b(-0.5-0)e^0 \]
\[ = 2.5a - 0.5b \]

Therefore,
\[ \begin{align*}
2.5a + b &= 10 \\
2.5a - 0.5b &= 0
\end{align*} \]
\[ \Rightarrow \quad a = 1.333, \quad b = 6.667 \]

The final solution is then
\[ U_1(t) = \left( 3.333e^{-0.2t} + 6.667e^{-0.5t} \right) \text{ V} \]
\[ U_2(t) = \left( 3.333e^{-0.2t} - 3.333e^{-0.5t} \right) \text{ V} \]

N.B.: Corrected lines are marked with an asterisk.