Lecture 24 - Frequency Response of Amplifiers (II)

Open-Circuit Time-Constant Technique

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Reading assignment:

Howe and Sodini, Ch. 10, §§10.4.4-10.4.5. 10.6
Key questions

• Is there a fast way to assess the frequency response of an amplifier?

• Do all amplifiers suffer from the Miller effect?
1. Open-Circuit Time-Constant Technique

Simple technique to estimate bandwidth of an amplifier.

Method works well if amplifier transfer function has:

- a dominant pole that dominates the bandwidth
- no zeroes, or zeroes at frequencies much higher than that of dominant pole

Transfer function of form:

\[
\frac{V_{out}}{V_s} = \frac{A_{vo}}{(1 + j\frac{\omega}{\omega_1})(1 + j\frac{\omega}{\omega_2})(1 + j\frac{\omega}{\omega_3})...}
\]

with

\[\omega_1 \ll \omega_1, \omega_2, \omega_3, ...\]
\[
\frac{V_{out}}{V_s} = \frac{A_{vo}}{(1 + j\frac{\omega}{\omega_1})(1 + j\frac{\omega}{\omega_2})(1 + j\frac{\omega}{\omega_3})...}
\]

Multiply out the denominator:

\[
\frac{V_{out}}{V_s} = \frac{A_{vo}}{1 + j\omega b_1 + (j\omega)^2 b_2 + (j\omega)^3 b_3...}
\]

where:

\[
b_1 = \frac{1}{\omega_1} + \frac{1}{\omega_2} + \frac{1}{\omega_3} + ...
\]

If there is a dominant pole, the low frequency behavior well described by:

\[
\frac{V_{out}}{V_s} \approx \frac{A_{vo}}{1 + j\omega b_1} = \frac{A_{vo}}{1 + j\frac{\omega}{\omega_H}}
\]

Bandwidth then:

\[
\omega_H \approx \frac{1}{b_1}
\]
It can be shown (see Gray & Meyer, 3rd ed., p. 502) that coefficient $b_1$ can be found exactly through:

$$b_1 = \sum_{i=1}^{n} \tau_i = \sum_{i=1}^{n} R_{Ti} C_i$$

where:

$\tau_i$ is open-circuit time constant for capacitor $C_i$

$R_{Ti}$ is Thevenin resistance across $C_i$

(with all other capacitors open-circuited)

Bandwidth then:

$$\omega_H \simeq \frac{1}{b_1} = \frac{1}{\sum_{i=1}^{n} \tau_i} = \frac{1}{\sum_{i=1}^{n} R_{Ti} C_i}$$
Summary of open-circuit time constant technique:

1. shut-off all independent sources
2. compute Thevenin resistance $R_{Ti}$ seen by each $C_i$ with all other $C$’s open
3. compute open-circuit time constant for $C_i$ as
   \[ \tau_i = R_{Ti} C_i \]
4. conservative estimate of bandwidth:
   \[ \omega_H \approx \frac{1}{\sum \tau_i} \]

Works also with other transfer functions: $\frac{I_{out}}{V_s}$, $\frac{V_{out}}{I_s}$, $\frac{I_{out}}{I_s}$. 
2. Application of OCT to evaluate bandwidth of common source amplifier

Small-signal equivalent circuit model (assuming current source has no parasitic capacitance):

Three capacitors ⇒ three time constants
□ First, short $v_s$:

\[ R_{Tgs} = R_S \]

and time constant associated with $C_{gs}$ is:

\[ \tau_{gs} = R_S C_{gs} \]
Time constant associated with $C_{db}$:

\[ R_{Td b} = R'_{out} \]

and time constant associated with $C_{gs}$ is:

\[ \tau_{gs} = R'_{out} C_{db} \]
Time constant associated with $C_{gd}$:

$$R_{Tgd} = R_S + R_{out}'(1 + g_m R_S) = R_{out}' + R_S(1 + g_m R_{out}')$$

and time constant associated with $C_{gd}$:

$$\tau_{gd} = \left[R_{out}' + R_S(1 + g_m R_{out}')\right]C_{gd}$$
The bandwidth is then:

\[
\omega_H \approx \frac{1}{\sum \tau_i} = \frac{1}{R_S C_{gs} + [R'_{out} + R_S(1 + g_m R'_{out})]C_{gd} + R'_{out} C_{db}}
\]

Identical result as in last lecture.

Open circuit time constant technique evaluates bandwidth neglecting \(-\omega^2\) term in the denominator of \(A_v\)

\(\Rightarrow\) conservative estimate of \(\omega_H\).
3. Frequency response of common-gate amplifier

Features:

- current gain \( \approx 1 \)
- low input resistance
- high output resistance
- \( \Rightarrow \) good current buffer
Small-signal equivalent circuit model:

\[ i_{out} = \frac{(g_m + g_{mb})v_{gs} + v_{gs}}{r_o + r_{oc}} \]

\[ R_L' = R_L + r_{oc} \]

□ Frequency analysis: first, open \( i_s \):
□ Time constant associated with $C_1$:

\[
\tau_1 = \left( C_{gs} + C_{sb} \right) \left( R_S / / R_{in} \right)
\]

Don’t need to solve:

- test probe is in parallel with $R_S$,
- test probe looks into input of amplifier ⇒ sees $R_{in}$!

\[
R_{T1} = R_S / / R_{in}
\]
Time constant associated with $C_2$:

\[
\tau_2 = (C_{gd} + C_{db})(R_L/\| R_{out})
\]

And:

\[
R_{T2} = R_L/\| R_{out}
\]

Again, don’t need to solve:

• test probe is in parallel with $R_L$,
• test probe looks into output of amplifier $\Rightarrow$ sees $R_{out}$!
\[ \omega_H \simeq \frac{1}{(C_{gs} + C_{sb})(R_S//R_{in}) + (C_{gd} + C_{db})(R_L//R_{out})} \]

No capacitor in Miller position → no Miller-like term.

Simplify:

- In a current amplifier, \( R_S \gg R_{in} \):
  \[ R_{T1} = R_S//R_{in} \simeq R_{in} \simeq \frac{1}{g_m + g_{mb}} \simeq \frac{1}{g_m} \]

- At output:
  \[ R_{T2} = R_L//R_{out} = R_L//r_{oc}//\{r_o[1+R_S(g_m+g_{mb}+\frac{1}{r_o})]\} \]
  or
  \[ R_{T2} \simeq R_L//r_{oc}//[r_o(1 + g_mR_S)] \simeq R_L \]

Then:

\[ \omega_H \simeq \frac{1}{(C_{gs} + C_{sb})\frac{1}{g_m} + (C_{gd} + C_{db})R_L} \]

If \( R_L \) is not too high, bandwidth can be rather high (and approach \( \omega_T \)).
Key conclusions

• Open-circuit time-constant technique: simple and powerful method to estimate bandwidth of amplifiers.

• Common-gate amplifier:
  – no capacitor in Miller position ⇒ no Miller effect
  – if $R_L$ is not too high, CG amp has high bandwidth

• $R_S, R_L$ affect bandwidth of amplifier