Lecture 5 - PN Junction and MOS
Electrostatics (II)

PN Junction in Thermal Equilibrium

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Reading assignment:

Howe and Sodini, Ch. 3, §§3.3-3.4
Key questions

• What happens if the doping distribution in a semiconductor abruptly changes from n-type to p-type?

• Is there a simple description of the electrostatics of a pn junction?
1. Introduction to pn junction

• pn junction: p-region and n-region in intimate contact

• Why is the p-n junction worth studying?

It is present in virtually every semiconductor device!

Example: CMOS cross section

Understanding p-n junction is essential to understanding transistor operation.
2. Electrostatics of p-n junction in equilibrium

Focus on intrinsic region:

Doping distribution of abrupt p-n junction:
What is the carrier concentration distribution in thermal equilibrium?

First think of two sides separately:

Now bring them together. What happens?

Diffusion of electrons and holes from majority carrier side to minority carrier side until drift balances diffusion.
Resulting carrier profile in thermal equilibrium:

- Far away from metallurgical junction: nothing happens
  - two quasi-neutral regions
- Around metallurgical junction: carrier drift must cancel diffusion
  - space-charge region
In a linear scale:

\[ p_o, n_o \]

\[ N_a - \]
\[ N_d + \]

Thermal equilibrium: balance between drift and diffusion

\[ \frac{J_{p}^{\text{diff}}}{J_{p}^{\text{drift}}} \]
\[ \frac{J_{n}^{\text{diff}}}{J_{n}^{\text{drift}}} \]

Can divide semiconductor in three regions:

- two quasi-neutral n- and p-regions (QNR’s)
- one space charge region (SCR)

Now, want to know \( n_o(x), p_o(x), \rho(x), E(x), \) and \( \phi(x) \).

Solve electrostatics using simple, powerful approximation.
3. The depletion approximation

- Assume QNR’s perfectly charge neutral
- assume SCR depleted of carriers (depletion region)
- transition between SCR and QNR’s sharp
  (must calculate where to place \(-x_{po}\) and \(x_{no}\))

\[ x < -x_{po} \quad p_o(x) = N_a, \quad n_o(x) = \frac{n_i^2}{N_a} \]

\[ -x_{po} < x < 0 \quad p_o(x), \quad n_o(x) \ll N_a \]

\[ 0 < x < x_{no} \quad n_o(x), \quad p_o(x) \ll N_d \]

\[ x_{no} < x \quad n_o(x) = N_d, \quad p_o(x) = \frac{n_i^2}{N_d} \]
• **SPACE CHARGE DENSITY**

\[ \rho(x) = \begin{cases} 
0 & x < -x_{po} \\
-qN_a & -x_{po} < x < 0 \\
qN_d & 0 < x < x_{no} \\
0 & x_{no} < x 
\end{cases} \]
• **Electric Field**

Integrate Gauss’ equation:

\[
E(x_2) - E(x_1) = \frac{1}{\varepsilon_s} \int_{x_1}^{x_2} \rho(x) dx
\]

\[\begin{align*}
\bullet \quad x < -x_{po} & \quad E(x) = 0 \\
\bullet \quad -x_{po} < x < 0 & \quad E(x) - E(-x_{po}) = \frac{1}{\varepsilon_s} \int_{-x_{po}}^{x} -qN_a dx = \frac{-qN_a}{\varepsilon_s} x \bigg|_{-x_{po}} = -qN_a \frac{(x + x_{po})}{\varepsilon_s} \\
\bullet \quad 0 < x < x_{no} & \quad E(x) = \frac{qN_d}{\varepsilon_s} (x - x_{no}) \\
\bullet \quad x_{no} < x & \quad E(x) = 0
\end{align*}\]
- **Electrostatic Potential**
  (with $\phi = 0$ @ $n_o = p_o = n_i$):

  \[
  \phi = \frac{kT}{q} \ln \frac{n_o}{n_i} \quad \phi = -\frac{kT}{q} \ln \frac{p_o}{n_i}
  \]

  In QNR’s, $n_o$, $p_o$ known $\Rightarrow$ can determine $\phi$:

  in p-QNR: $p_o = N_a \Rightarrow \phi_p = -\frac{kT}{q} \ln \frac{N_a}{n_i}$

  in n-QNR: $n_o = N_d \Rightarrow \phi_n = \frac{kT}{q} \ln \frac{N_d}{n_i}$

\[
\begin{align*}
\phi & = \phi_n - \phi_p = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2} \\
\phi_B & = \phi_n - \phi_p = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2}
\end{align*}
\]

General expression: did not use depletion approximation.
To get $\phi(x)$ in between, integrate $E(x)$:

$$\phi(x_2) - \phi(x_1) = -\int_{x_1}^{x_2} E(x) \, dx$$
\begin{itemize}
  \item $x < -x_{po}$ \hspace{1cm} $\phi(x) = \phi_p$
  \item $-x_{po} < x < 0$ \hspace{1cm} $\phi(x) - \phi(-x_{po})$
    \[= - \int_{-x_{po}}^{x} \frac{qN_a}{\epsilon_s} (x + x_{po}) \, dx\]
    \[= \frac{qN_a}{2\epsilon_s} (x + x_{po})^2\]
    \[\Rightarrow \phi(x) = \phi_p + \frac{qN_a}{2\epsilon_s} (x + x_{po})^2\]
  \item $0 < x < x_{no}$ \hspace{1cm} $\phi(x) = \phi_n - \frac{qN_d}{2\epsilon_s} (x - x_{no})^2$
  \item $x_{no} < x$ \hspace{1cm} $\phi(x) = \phi_n$
\end{itemize}

Almost done...
Still don’t know $x_{no}$ and $x_{po}$ $\Rightarrow$ need two more equations

1. Require overall charge neutrality:

$$qN_ax_{po} = qN_dx_{no}$$

2. Require $\phi(x)$ continuous at $x = 0$:

$$\phi_p + \frac{qN_a}{2\varepsilon_s}x_{po}^2 = \phi_n - \frac{qN_d}{2\varepsilon_s}x_{no}^2$$

Two equations with two unknowns. Solution:

$$x_{no} = \sqrt{\frac{2\varepsilon_s\phi_B N_a}{q(N_a + N_d)N_d}}$$
$$x_{po} = \sqrt{\frac{2\varepsilon_s\phi_B N_d}{q(N_a + N_d)N_a}}$$

Now problem completely solved.
Other results:

Total width of space charge region:

\[ x_{do} = x_{no} + x_{po} = \sqrt{\frac{2\epsilon_s \phi_B (N_a + N_d)}{qN_a N_d}} \]

Field at metallurgical junction:

\[ |E_o| = \sqrt{\frac{2q\phi_B N_a N_d}{\epsilon_s (N_a + N_d)}} \]
Three cases:

- Symmetric junction: \( N_a = N_d \Rightarrow x_{po} = x_{no} \)

- Asymmetric junction: \( N_a > N_d \Rightarrow x_{po} < x_{no} \)

- Strongly asymmetric junction: 
  *i.e.* \( p^+n \) junction: \( N_a \gg N_d \)

\[
x_{po} \ll x_{no} \approx x_{do} \approx \frac{2\epsilon_s \phi_B}{qN_d} \propto \frac{1}{\sqrt{N_d}}
\]

\[
|E_o| \approx \sqrt{\frac{2q\phi_B N_d}{\epsilon_s}} \propto \sqrt{N_d}
\]

The lowly-doped side controls the electrostatics of the pn junction.
4. Contact potentials

Potential distribution in thermal equilibrium so far:

![Diagram showing potential distribution](image)

Question 1: *If I apply a voltmeter across diode, do I measure $\phi_B$?*

- ☐ yes
- ☐ no
- ☐ it depends

Question 2: *If I short diode terminals, does current flow on outside circuit?*

- ☐ yes
- ☐ no
- ☐ sometimes
We are missing *contact potential* at metal-semiconductor contacts:

\[
\phi_B = \phi_{mn} + \phi_{mp}
\]
Key conclusions

• Electrostatics of pn junction in equilibrium:
  – a space-charge region
  – surrounded by two quasi-neutral regions
⇒ built-in potential across p-n junction

• To first order, carrier concentrations in space-charge region are much smaller than doping level
⇒ depletion approximation.

• Contact potential at metal-semiconductor junctions:
⇒ from contact to contact, there is no potential build-up across pn junction