1. (Problem 6.11) Consider the Markov chain shown below. Let us refer to a transition that results in a state with a higher (respectively, lower) index as a birth (respectively, death). Calculate the following quantities, assuming that when we start observing the chain, it is in steady state.

\[
\begin{array}{ccc}
  & 0.4 & 0.6 \\
 1 & 0.3 & \\
  & 0.5 & 0.2 \\
 2 & 0.2 & \\
  & 0.8 & \\
 3 & & \\
\end{array}
\]

(a) For each state \(i\), the probability that the current state is \(i\).
(b) The probability that the first transition we observe is a birth.
(c) The probability that the first change of state we observe is a birth.
(d) The conditional probability that the process was in state 2 before the first transition that we observe, given that this transition was a birth.
(e) The conditional probability that the process was in state 2 before the first change of state that we observe, given that this change of state was a birth.
(f) The conditional probability that he first observed transition is a birth given that it resulted in a change of state.
(g) The conditional probability that the first observed transition leads to state 2, given that it resulted in a change of state.

2. Consider the Markov chain below. For all parts of this problem the process is in state 3 immediately before the first transition. Be sure to comment on any unusual results.
(a) Find the variance for $J$, the number of transitions up to and including the transition on which the process leaves state $S_3$ for the last time.

(b) Find $\pi_i$ for $i = 1, 2, \ldots, 4$, the probability that the process is in state $i$ after $10^{10}$ transitions or explain why these probabilities can’t be found.

(c) Given that the process never enters state 4, find the $\pi_i$’s or explain why they can’t be found.