Fold & cut software [David Benjamin & Anthony Lee 2010]
  - DEMO (6.849 project)
  - PROJECT: improve UI, make Java applet; port to JavaScript; force degeneracies; or compute folded state & build/unfold
  - JOrigami: disks [Silveira, Cosentino, Coelho, Aoki]

Odd-degree vertices?
- even degree $\iff$ face 2-colorable
  $\iff$ alternating above/below side assignment
  $\iff$ uncreased cut edges
  $\iff$ scissor cuts (separate material on both sides of line)
- mathematical/laser cuts (removing line) can do odd-degree vertices
  e.g.

- if graph doesn’t disconnect from the removal of any 1 edge (planar 2-edge-connected) then = union of two even graphs
  [Demaine, Demaine, Lubiw 1998, thanks to Jim Geelen & Dan Younger]
- Linear corridors $\rightarrow$ tree
  - corridor $\rightarrow$ edge (or ray) [flap]
  - width $w$ $\rightarrow$ length $w$
  - perpendicular $\rightarrow$ vertex [hinge]
    (connected comps)
  - similar to TreeMaker CP $\rightarrow$ shadow tree

- Tree folding $\rightarrow$ origami folding
  - expand each edge to accordion folding
  - stitch together at perpendiculars

- Irrational ratio happens with prob. 1? YES
  - but first need closed loop of perpendiculars
    - CONJECTURE: with prob. 1, only get loops around one cut vertex
      (normal circular corridor)

- e.g.

- Examples: students & HELL

- Disk packing $\rightarrow$ tri/quad decomposition
  - disk center $\rightarrow$ vertex
  - kissing disks $\rightarrow$ edge
  - 3- or 4-gap $\rightarrow$ triangle or quad.
0 How many disks? $\mathcal{O}\left(\sum_{x \in \mathcal{P}} \frac{dx}{lfs(x)}\right)$
- $lfs(x) = \frac{\text{local feature size}}{\text{radius of smallest disk centered at } x}$
  hitting a nonincident edge of $\mathcal{P}$

0 Disk packing method vs. tree method
- disks
- easy to place (but many)
- input = polygon
- regions = tri. & quad.
- both align boundaries of universal molecules
- disks & rivers
- hard to place
- input = tree
- regions = convex (or tri.)

0 Straight skeleton method vs. tree method
- arbitrary polygons/graphs
- no control on tree/lengths
- $\text{POLYGON PACKING} \approx \text{combination of two}$
  (straight skeleton + gussets to control)
  [Demaine, Demaine, Lang] [Origami Design Secrets 2e]

0 OPEN: fold flat & cut of fixed curvature $k$
- make all unions of arcs of this curvature?
  $- \text{intuition: } \frac{1}{k}$
  $\text{arc} \rightarrow \text{fold & cut}$
- but:
- Flattening
  - 3D fold & cut $\Rightarrow$ flat folded state (folding motions not preserved)
  - **NEW**: convex polyhedra can be continuously flattened [Itoh, Nara, Vilcu 2011]
  - **PROJECT**: animate their motion
  - **OPEN**: nonconvex polyhedra?

**PROJECT**: fold & cut alphabet
e.g. 3 or 4 simple folds/letter or CP for entire word/page

**PROJECT**: paper cutting art via fold & cut (à la Peter Callesen)
6.849 Geometric Folding Algorithms: Linkages, Origami, Polyhedra
Fall 2012

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