0 (at end) Finish Hypar Truncated Tetrahedron

0 Folded states → folding motions: what's wrong with holes?
- could try to fill hole
- but folded state doesn't define where hole goes
- might be impossible without intersection
- e.g.

- possible (even with finite # creases)
- impossible with "holes" filled...

0 Sliding joints in linkages
- can be simulated by regular linkages via Peaucellier:
Kempe Universality Theorem:

- Why unbraced contra/parallelogram bad?
- How contraparallelogram bracing works \[\text{[Abbott & Barton 2004]}\]

\(- |xpl| = |xrl| \& |xs| = |xq|
\Rightarrow x \text{ lies on perp. bisectors of } pr \& sq
\Rightarrow x \text{ would lie interior to parallelogram (actually center)}
\Rightarrow \text{impossible for } |xpl| > \text{perimeter}
- placing x in contraparallelogram:

\[
|spl| \cdot |srl| = \frac{1}{4} (|abl|^2 - |ald|^2)
\]

\[
|xs|^2 - |xpl|^2 = |spl| \cdot |srl|
\]

\Rightarrow \text{set } |xs|^2 = |xpl|^2 + \frac{1}{4} (|abl|^2 - |ald|^2)

PROJECT:
- implement Kempe eg. for splines
- "Kempe" alphabet
- Kempe sculpture
Generalization: [Abbott, Barton, Demaine 2008]

1. Higher dimensions:
   - 3D Peaucellier restricts to plane
   - two restrict to line
   - Kempe construction in xy plane
   - copy angles/lengths into/out of xy

2. Semi-algebraic set = finite union/intersection of polynomial inequalities \( p(x,y) \geq 0 \)
   - includes splines (piecewise polynomial)
   - inequality by modified Peaucellier:

   - intersection: overlay two linkages at \( p \)
   - union:

   \[
   \begin{align*}
   L_1 & : \quad \left[ (x-x_1)^2 + (y-y_1)^2 \right] \cdot \\
   L_2 & : \quad \left[ (x-x_2)^2 + (y-y_2)^2 \right] = 0
   \end{align*}
   \]
Axioms:

- straight edge & compass:
  1. can compute $\emptyset, 1, +, -, \times, \div$, $\sqrt{\_}$, $\sqrt[3]{\_}$ (⇒ solve quadratics) & that's all
  2. can't trisect $60^\circ$ or compute $\sqrt[3]{2}$ [Wenzel 1837]

- single-fold origami: [Huzita 1989; Hatori 2002; Justin 1989; Lang 2010]
  1. can compute $\emptyset, 1, +, -, \times, \div, \sqrt{\_}, \sqrt[3]{\_}$ (⇒ solve cubics & quartics) & that's all
  3. can trisect angles but can't quintisect [Abe]
  4. can compute $\sqrt[3]{2}$ [Messer 1985]
  5. Reference Finder software [Lang]

- two-fold origami [Alperin & Lang - OSME 2006]
  - can quintisect angles

- n-fold origami [Alperin & Lang; Demaine & Demaine 2000]
  - can solve any polynomial