Hinged dissection software: just specific examples

**PROJECT:** hinged dissection animator
- implement slender adornments (refinement + expansive motion)
- implement general algorithm?
- implement polyform algorithm

**PROJECT:** design elegant hinged dissections

Polyform = n copies of one shape glued together along corresp. edges

Inductive construction:
- **base case:** hinge-dissect 1 copy such that every edge has incident hinge
- **step:** take spanning tree of copies remove leaf copy induct on n-1 remaining copies rotate base case to meet them reconnect ~ get same hinging

⇒ folded states (use slender for motion)

Also: polyΔ → poly□, etc.
3D [Demaine, Demaine, Lindy, Souvaine 2005]

Physical:
- in liquid
- DNA
- Macrobot/Decibot
- related: reconfigurable robots

- Rectangle \rightarrow rectangle [Montucla 1778]
  - superposing strips method

- same method for Dudeney's $\Delta \rightarrow \Box$
- more stable table [Frederickson 2008]

PROJECT: build reconfigurable furniture

- \# pieces doubles? at least, in worst case
Pseudopolynomial: say integer

if polygon vertices lie on common grid,
# pieces = poly(n, r)
→ # grid positions = \frac{\text{size}}{\text{cell size}}

- idea: ensure constant-depth recursion
  ① triangulate polygons with grid vertices
     ⇒ matching Δ areas of 1/2 [Pick's Theorem]
  ② chainify \( \triangle \) \( \Rightarrow \) \( \triangledown \)
     ⇒ vertices on \( \frac{1}{3} \) grid
  ③ fix which vertices connect which Δs
     by only modifying parent in subtree move
  ④ Δ \( \Rightarrow \) Δ by overlaying 3 constructions:
    A
    B
    C
    \leftarrow \text{actually done last}

... using pseudocuts
    \Rightarrow \text{simulate cut overlays}
3D dissection:
- volumes must match
- insufficient by Dehn's solution [1901]
to Hilbert's Third Problem [1900]
- Dehn invariants must match:
  \[ \sum_{\text{edge } e} l(e) \otimes \left[ \Theta(e) + \mathbb{Q} \cdot \pi \right] \]
  ignore added rational multiples of \( \pi \Rightarrow \text{"irrational part"} \)
- tensor product space: linear combination of pairs \( l \otimes \Theta \) where
  \[ l_1 \otimes \Theta + l_2 \otimes \Theta = (l_1 + l_2) \otimes \Theta \]
  \[ l \otimes \Theta_1 + l \otimes \Theta_2 = l \otimes (\Theta_1 + \Theta_2) \]
  \[ c (l \otimes \Theta) = (cl) \otimes \Theta = l \otimes (c \Theta) \quad \forall c \in \mathbb{Q} \]
- Dehn's Theorem: invariant under dissection
  - e.g.: cut edge \( (l_1 + l_2) \otimes \Theta \rightarrow l_1 \otimes \Theta + l_2 \otimes \Theta \)
  - slice angle \( l \otimes (\Theta_1 + \Theta_2) \rightarrow l \otimes \Theta_1 + l \otimes \Theta_2 \)
  \( \Rightarrow \) no dissection of cube \( \rightarrow \) regular tetrahedron

\[ 12 (1 \otimes 90^\circ) = \emptyset \]
\[ 6 (2.04\ldots \otimes 70.5288\ldots) \]
\[ \arccos \left( \frac{1}{3} \right) \]
- 3D dissection exists ⇔ volumes & Dehn Invariants match [Sydler 1965]  
  - ditto in 4D [Jessen 1968]  

**OPEN**: 5D & higher?  

**OPEN**: efficient algorithm to check Dehn match  
  - decidable [Kreinovich-Geomb, 2008]  

**OPEN**: algorithm to find dissection  
  - refinement into hinged dissection still works [Abel et al.]